INSTRUCTIONS TO CANDIDATES -

1. On the answer sheet, make all the entries carefully in the space provided **ONLY** in **BLOCK CAPITALS** as well as by properly darkening the appropriate bubbles.

   Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.

2. On the answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling the bubbles.

3. Question paper has 80 multiple choice questions. Each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

   Q. No. 22  a  b  c  d

4. A correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer.

5. Any rough work should be done only in the space provided.

6. No candidate should leave the examination hall before the completion of the examination.
1. A loudspeaker emits sound at a maximum audible level of 130 dB when measured directly from a distance of 1 metre. If the safe limit of audible sound to our ears is 90 dB, a listener must stand directly at a minimum distance of (a) 1.44 m (b) \( e^2 \) m (c) 100 m (d) 2.09 m

Answer (c)

Sol. \( \text{L} = 10 \log_{10} \left( \frac{I}{I_0} \right) \text{ dB} \)

Given 130 = 10 \( \left[ \log \left( \frac{k}{(1)^2} \right) - \log I_0 \right] \) \( \cdots (i) \)

Let listener must stand at distance \( r \)

90 = 10 \( \left[ \log \left( \frac{k}{(r)^2} \right) - \log I_0 \right] \) \( \cdots (ii) \)

Equation (i) – Equation (ii)

\( \Rightarrow 40 = 10[2\log_{10} r - 0] \)

\( \Rightarrow r = 100 \text{ m} \)

2. The diameter of radio telescope, working at a wavelength of \( \lambda = 1 \text{ cm} \), with the same resolution as optical telescope of diameter \( D = 10 \text{ cm} \)

(a) 2 m (b) 2 km (c) 20 km (d) 200 km

Answer (b)

Sol. \( \frac{1.22 \lambda}{D} = \frac{\lambda_{\text{visible}} \times 1.22}{10 \times 10^{-2}} \)

\( D = \frac{\lambda}{\lambda_{\text{visible}}} \times 0.1 \)

\( D = \frac{10^{-3} \times 10^{10}}{5000} = \frac{10^4}{5} \)

\( D = 2 \text{ km} \)

3. In a binary system, the apparent magnitude of the primary star is 1.0 and that of the secondary star is 2.0. The maximum combined magnitude of this system is (a) 3 (b) 1.5 (c) 1 (d) 064

Answer (d)

Sol. Let \( I_1 \) be the luminosity of star 1, then \( m_1 \) be the apparent magnitude so \( m_1 = -2.5 \log \left( \frac{I_1}{I_0} \right) \)

and so, \( 1 = -2.5 \log \left( \frac{I_1}{I_0} \right) \) \( \Rightarrow \frac{2}{5} = \log \left( \frac{I_1}{I_0} \right) \)

\( \Rightarrow I_1 = I_0 \times 10^{-2/5} \) similarly for \( m_2 = 2 \), we get \( I_2 = I_0 \times 10^{-4/5} \)

Now for combined system let \( m \) be the magnitude then

\( m = -2.5 \log \left( \frac{I_1 + I_2}{I_0} \right) \)

\( \Rightarrow I_1 + I_2 = I_0 \times 10^{-2m/5} \)

\( \Rightarrow \frac{1}{10^{-2/5}} + \frac{1}{10^{-4/5}} = \frac{1}{10^{2m/5}} \)

\( \Rightarrow 10^{2m/5} = \frac{10^{6/5}}{10^{4/5} + 10^{2/5}} \)

\( \Rightarrow 10^{2m/5} = 1.796 \)

\( \Rightarrow m = 0.636 \approx 0.64 \)

4. Suppose the tangent to the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) at the point \( \left( \frac{ae}{a^2}, \frac{-b^2}{a} \right) \) makes an angle of 30° with x-axis. Then \( \frac{b^2}{a^2} \) equals

(a) \( \frac{1}{3} \) (b) \( \frac{1}{2} \) (c) \( \frac{2}{3} \) (d) \( \frac{3}{4} \)

Answer (c)

Sol. Equation of tangent : \( \frac{x(ae)}{a^2} + \frac{y(\frac{b^2}{a})}{b^2} = 1 \)

\( \Rightarrow y = xe - a \)

\( \therefore \) slope = \( e = \tan 30° \)

\( \Rightarrow e^2 = \frac{1}{3} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{3} \)

\( \therefore \frac{b^2}{a^2} = \frac{2}{3} \)
5. A piece of strong magnet is suspended from a helical spring made of a non magnetic material and oscillates in a vertical plane with a time period of T on the surface of the earth. If this is taken to the moon then it will oscillate
(a) With a time period \( T_1 > T \) as the value of 'g' is smaller on the moon
(b) With a time period \( T_1 < T \) as the value of 'g' is smaller on the moon
(c) With a time period \( T_1 < T \) as there is no magnetic field on the moon
(d) With the same time period as the spring and the suspended body are the same on the moon

Answer (d)

Sol. \[ T = 2\pi \sqrt{\frac{m}{k}} \]

Time period depends on spring constant and mass m
Here, \( T' = T \)
Time period will be same on earth as well as moon.

6. The number of triples \((a, b, c)\) of natural numbers satisfying the equation \[ \frac{5}{12} = \frac{1}{a} + \frac{1}{ab} + \frac{1}{abc} \]

(a) 7
(b) 8
(c) 9
(d) 12

Answer (a)

Sol. \[ \left( \frac{5a}{12} - 1 \right)b - 1 \]

\[ a > 2 \] else \( \frac{5a}{12} - 1 \) is -ve

\[ a < 7 \] else \( \left( \frac{5a}{12} - 1 \right)b - 1 > 1 \), forcing c to be less than 1.

Case: \( a = 6 \)

\[ \left( \frac{3b}{2} - 1 \right)c = 1 \Rightarrow (b, c) = (1, 2) \]

Hence, total 7 solutions.

7. A 1.5 times magnified real image of an object is obtained when it is placed 16 cm away from a thin convex lens. Now a thin concave lens is placed in contact with the convex lens keeping the object undisturbed and an image of same magnification is formed by the combination. The focal length of the concave lens is
(a) 8 cm
(b) 10 cm
(c) 12 cm
(d) 16 cm

Answer (c)

Sol. Clearly, \[ |m| = \left| \frac{v}{u} \right| = 1.5 \]

For convex lens (for real image)
\[ v = + 24 \text{ cm}, \quad u = - 16 \text{ cm} \]

\[ \frac{1}{f_1} = \frac{1}{24} + \frac{1}{16} = \frac{5}{48} \quad \text{...(i)} \]

Now for combination of convex and concave lens,
effective focal length would change. It means we cannot have real image of same magnification. Or we shall have the virtual image.

\[ u = - 16 \text{ cm}, \quad v = - 24 \text{ cm} \]

\[ \Rightarrow \frac{1}{f_{eq}} = \frac{1}{-16} + \frac{1}{24} = \frac{1}{48} \]

Now, \[ \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} \]

\[ \Rightarrow \frac{1}{f_2} = \frac{1}{48} \]

\[ \Rightarrow f_2 = - \frac{48}{48} \]

\[ \Rightarrow f_{2l} = - 12 \text{ cm} \]

Hence, total 7 solutions.
8. Let ABC be an equilateral triangle with side x. Two points P and Q are inside ABC such that PQ is parallel to BC and AP = AQ = PB = QC = \( \frac{1}{\sqrt{3}} + 1 \) and PQ = \( \sqrt{2} \). Then x equals

(a) \( 4\sqrt{2} + 2\sqrt{6} \)  
(b) \( 2\sqrt{2} + \sqrt{6} \)  
(c) \( 2\sqrt{3} + \sqrt{6} \)  
(d) \( 2\sqrt{6} + \sqrt{3} \)

Answer (b)

\[ \text{Sol.} \]

\[ \begin{align*}
180° - 2\theta + 2\alpha &= 60° \\
\Rightarrow \theta - \alpha &= 60°
\end{align*} \]

\[ 2\text{AP cos}\theta = \sqrt{2} \quad \Rightarrow \cos\theta = \frac{1}{\sqrt{2}(\sqrt{3} + 1)} = \frac{\sqrt{3} - 1}{2\sqrt{2}} \\
\Rightarrow \sin\theta = \frac{\sqrt{3} + 1}{2\sqrt{2}} \\
x = 2\text{AP cos}\alpha = 2\text{AP cos}(\theta - 60°) = 2\text{AP}[\cos\theta \cos60° + \sin\theta \sin60°] \\
= 2(\sqrt{3} + 1) \left[ \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{1}{2} + \frac{\sqrt{3} + 1}{2\sqrt{2}} \times \frac{1}{2} \right] \\
= 2\sqrt{3} + \sqrt{6}
\]

9. Critical velocity, drift velocity, escape velocity and rms velocity are the different types of velocities that we come across in the same order while discussing

(a) Viscosity, electron motion in solids, gravitation, surface tension respectively
(b) Motion of gas molecules, viscosity, gravitation, electron motion in solids respectively
(c) Sound propagation, gravitation, motion of gas molecules, colour of light respectively
(d) Viscosity, electron motion in solids, gravitation, motion of gas molecules respectively

Answer (d)

\[ \text{Sol.} \]

\[ \begin{align*}
\angle \text{FAD} &= 120° \\
\text{ar}(\triangle \text{FAD}) &= \frac{1}{2} \times \text{AF} \times \text{AD} \sin 120° = \frac{1}{2} \times \frac{\sqrt{3}}{2} 5k^2 \\
\text{ar}(\triangle \text{ABC}) &= \text{ar}(\triangle \text{ABC}) + 3\text{ar}(\triangle \text{FAD}) \\
\Rightarrow \text{ar}(\triangle \text{FED}) &= \frac{\sqrt{3}}{4} \times (4k)^2 + \frac{3\sqrt{3}}{4} \times 5k^2 \\
\text{ar}(\triangle \text{DEF}) &= \frac{31}{16}
\end{align*} \]
11. Sun is at a mean distance of about 27,000 light years from the centre of the Milky way galaxy and completes one revolution about the galactic centre in about 225 million years. The linear speed of Sun is
(a) 160 km s\(^{-1}\)  
(b) 230 km s\(^{-1}\)  
(c) 30 km s\(^{-1}\)  
(d) 80 km s\(^{-1}\)
Answer (b)

Sol. Angular velocity \(\omega\) \(=\) \(\frac{2\pi}{225 \times 10^6 \times (1 \text{ year})}\)
\[\omega = \frac{2\pi}{225 \times 10^6 \times (365 \times 24 \times 3600)} \text{ rad/s}\]

\[R = 27,000 \times 3 \times 10^8 \times (365 \times 24 \times 3600)\]
\[V = \omega R\]
\[\Rightarrow V = \frac{2\pi \times 27,000 \times 3 \times 10^8 \times (365 \times 24 \times 3600)}{(365 \times 24 \times 3600) \times 225 \times 10^6}\]
\[= \frac{2\pi \times 27,000 \times 3 \times 10^8}{225 \times 10^6} = 226.2 \text{ km/s}\]

If we use \(C = 2.99 \times 10^8 \text{ m/s}\)
Closest answer is 230 km/s

12. Light from the nearest star ‘proxima centauri’ takes 4.24 light years to reach earth. The stellar parallax of this star is about
(a) 1.30 s \(\quad\) (b) 0.77 s \(\quad\) (c) 13.8 s \(\quad\) (d) 0.24 s
Answer (b)

Sol. Stellar parallax is measured in arc/seconds
\[P = \frac{1}{d} \quad (d \text{ is the distance in parsec})\]
\[P = \frac{1 \times 3.26}{4.24} = 0.77 \text{ arc/second}\]

13. A block of conductor with its area equal to ‘A’ and thickness ‘b’ is placed between the plates of a parallel plate capacitor without touching either of the plates. If the area of the plates of the capacitor be ‘A’ each and ‘d’ be the separation between the plates then the capacitance of the system after the introduction of the block is
(a) \(\frac{\varepsilon_0 A}{d}\) \(\quad\) (b) \(\frac{\varepsilon_0 A}{d \left(1 + \frac{b}{d}\right)}\)
(c) \(\frac{\varepsilon_0 A}{d \left(1 - \frac{b}{d}\right)}\) \(\quad\) (d) \(\frac{\varepsilon_0 A}{d \left[1 + \left(\frac{b}{d}\right)^2\right]}\)
Answer (c)

Sol. Capacitance of remaining portion is
\[\frac{\varepsilon_0 A}{(d - b)} = C_{eq}\]

14. The number of real solutions of the equation
\[|x - x - |x - 4|| = x^2 - 4x\]
is
(a) 0 \(\quad\) (b) 1 \(\quad\) (c) 2 \(\quad\) (d) More than 2
Answer (c)

Sol. \(x \geq 4\)
\[|x - x - (x - 4)| = x^2 - 4x\]
\[\Rightarrow x - 4 = x (x - 4)\]
\[\Rightarrow x = 4\]

\(2 \leq x < 4\)
\[|x - 2x - 4| = x^2 - 4x\]
\[\Rightarrow |x + 4| = x^2 - 4x \Rightarrow 4 - x = x(x - 4)\]
No solution

\(\frac{4}{3} \leq x < 2\)
\[|x - 2x - 4| = x^2 - 4x\]
\[\Rightarrow |x - (4 - 2x)| = x^2 - 4x\]
\[\Rightarrow 3x - 4 = x^2 - 4x \Rightarrow x^2 - 7x + 4 = 0\]
\[x^2 - 7x + 4\] is negative at \(x = \frac{4}{3}, 2 \Rightarrow\) No solution
\[x < \frac{4}{3}\]
\[4 - 3x = x^2 - 4x \Rightarrow x^2 - x - 4 = 0\]
15. A body of mass 1.0 kg is pulled along a rough horizontal surface by a horizontal force of 5 N for 10 s starting from rest. If the kinetic friction is $\mu_k = 0.40$, the amount of heat generated is equal to (assuming $g = 10 \text{ ms}^{-2}$)

(a) 190 J  
(b) 200 J 
(c) 210 J  
(d) 205 J

Answer (b)

Sol. 

$$a = \frac{5 - (0.4) \times 1 \times 10}{1} = 1 \text{ m/s}^2$$

$$S = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 1 \times (10)^2 = 50 \text{ m}$$

$$H = W_{k_f} = (50) \times (0.4) \times 10 = 200 \text{ J}$$

16. Six dice are rolled simultaneously. The probability of getting at least four identical numbers is

(a) $\frac{2250}{6^6}$  
(b) $\frac{2436}{6^6}$  
(c) $\frac{2535}{6^6}$  
(d) $\frac{2738}{6^6}$

Answer (b)

Sol. No. of outcomes when 4 are identical of one type and 2 are identical of other type $= ^6C_2 \times ^2C_1 \times \frac{6!}{4!2!} = 450$

No. of outcomes when 4 are identical of one type and rest 2 are distinct $= ^6C_2 \times ^5C_1 \times \frac{6!}{4!} = 1800$

No. of outcomes when 5 are identical and 1 different $= ^6C_2 \times ^2C_1 \times \frac{6!}{5!} = 180$

No. of outcomes when all are identical = 6

Required probability $= \frac{450 + 1800 + 180 + 6}{6^6} = \frac{2436}{6^6}$

17. The ceiling of a long hall is 45 m high. The maximum horizontal distance that a ball thrown with a speed of 50 ms$^{-1}$ can go without hitting the ceiling is nearly equal to ($g = 10 \text{ ms}^{-2}$)

(a) 250 m  
(b) 240 m  
(c) 230 m  
(d) 300 m

Answer (b)

Sol.

Given $u_x^2 + u_y^2 = (50)^2$ ...(1)

Also $\frac{v_y^2}{2g} = 45$

$\Rightarrow u_y = 30 \text{ m/s}$

From equation (i) $u_x^2 = 1600$

$u_x = 40 \text{ m/s}$

$R_{max} = (40) \times \frac{2 \times 30}{10} = 240 \text{ m}$

18. The tangents drawn from a certain point $P$ to the parabola $2y = x^2 - 2$ are also tangents to the parabola $4y = x^2 - 10x + 37$. The sum of the coordinates of $P$ is

(a) 10  
(b) 6  
(c) 0  
(d) $-10$

Answer (d)

Sol. $x^2 = 2(y + 1)$ and $4(y - 3) = (x - 5)^2$ are the given parabola

Tangents to them are

$$y + 1 = mx - \frac{1}{2}m^2$$  

... (i)

$$y - 3 = m(x - 5) - m^2$$  

... (ii)

(i) $\&$ (ii) $\Rightarrow m^2 + 10m - 8 = 0 \Rightarrow m = -5 \pm \sqrt{33}$

(i) $- (ii) \Rightarrow 0 = (m_1 - m_2)x - \frac{1}{2}(m_1^2 - m_2^2)$

$\Rightarrow x = \frac{m_1 + m_2}{2} = -5$
\[ m_2 \times (i) - m_1 \times (ii) \Rightarrow y(m_2 - m_1) + (m_2 - m_1) \]
\[ = -\frac{1}{2}m(m_1 - m_2) \]
\[ \Rightarrow y + 1 = \frac{m(m_2)}{2} = -4 \Rightarrow y = -5 \]
\[ \therefore x + y = -10 \]

19. A yo-yo of mass 'M' and radius of the inner hub 'r' is completely wound with a string. It is allowed to start unwinding with zero downward initial velocity. The moment of inertia of the yo-yo about an axis passing through its centre of mass and normal to the discs is \( I \). The acceleration with which the yo-yo falls when \( I = Mr^2 \) can be given by

(a) \( a = g \)  
(b) \( a = g/2 \)  
(c) \( a = 2g/3 \)  
(d) \( a = g/4 \)

Answer (b)

\[ \text{Answer (d)} \]

\[ \text{Let PQ be parallel to base BC.} \]
\[ \text{PQ = 3x, AP = 4x} \]
\[ \text{Area (\( \Delta APQ \))} = \frac{1}{2} \times 3x \times 4x \]
\[ = \frac{1}{2} \text{Area (\( \Delta ABC \))} \]
\[ \Rightarrow 6x^2 = 3 \Rightarrow \frac{x^2}{\frac{1}{2}} = \frac{1}{\sqrt{2}} \]
\[ \therefore \text{PQ =} 3x = \frac{3\sqrt{2}}{2} = \frac{3 \times 1.414}{2} = 4.242 \]
\[ = 2.121 \]
\[ \therefore \text{So the least possible length} \leq 2.121 \]
\[ \therefore \text{Only possible option is option (d)} \]

20. The wave length of H\( _\alpha \) line from hydrogen discharge tube in a laboratory is 656 nm. The corresponding radiation received from two galaxies A and B have wavelengths of 648 nm and 688 nm respectively. Then

(a) A is approaching the earth with a speed of \( 2.4 \times 10^4 \) kms\(^{-1} \)  
(b) B is approaching the earth with a speed of \( 1 \times 10^4 \) kms\(^{-1} \)  
(c) A is receding from the earth with a speed of \( 3.6 \times 10^4 \) kms\(^{-1} \)  
(d) B is receding the earth with a speed of \( 1.5 \times 10^4 \) kms\(^{-1} \)

Answer (d)

\[ \text{Sol. By Doppler's shift in light} \]
\[ \frac{\Delta \lambda}{\lambda} = \frac{V}{c} \]

For galaxy A
\[ \frac{-(648 - 656)}{656} = \frac{V}{3 \times 10^8} \]
\[ V = 3.6 \times 10^6 \text{ m/s or } 3.6 \times 10^3 \text{ km/s (approaching)} \]
For galaxy B

\[-\frac{(688 - 656)}{656} = \frac{V}{3 \times 10^9}\]

\[V = -1.5 \times 10^7 \text{ m/s or } -1.5 \times 10^4 \text{ Km/s (receding)}\]

22. The correct sequence of the objects in the ascending order of distance from the sun, is

(a) Kupier belt, Uranus, Asteroid belt and oort cloud
(b) Uranus, Asteroid belt, Oort cloud and Kupier belt
(c) Oort cloud, Asteroid belt, Uranus and Kupier belt
(d) Asteroid belt, Uranus, Kupier belt, and Oort cloud

Answer (d)

Sol. Distance from Sun

Kupier belt = 50 AU
Uranus = 19.2 AU
Asteroid belt = 3.2 AU
Oort cloud = 50000 AU

\[\therefore \text{ Asteroid belt < Uranus < Kupier belt < Oort cloud}\]

23. A cone of height \(h\) is floating in a liquid upside down with a mass \(m\) attached to it as shown in the figure. Water reaches a height of \(h/2\) at equilibrium. The cone is now given a small downward push and is found to oscillate about its mean position. If friction is ignored the frequency of this oscillation is

\[m \frac{h}{2} \rho g = mg \quad \ldots (i)\]

\[\text{from triangle property}\]

\[\frac{R}{r} = \frac{h}{h/2}\]

\[\Rightarrow r = \frac{R}{2}\]

Equation (i) will become

\[mg = \frac{\pi R^2 h}{24} \rho g\]

\[m = \frac{\pi R^2 h \rho}{24}\]

Now, additional buoyancy force is restoring force. So,

\[\pi R^2 \rho g - \pi \frac{h^2}{2} \rho g = ma\]

\[a = \frac{-\pi R^2 \rho g}{4} - \frac{\pi R^2 h \rho}{24} \times \frac{x}{x}\]

\[\Rightarrow \omega^2 = \frac{6g}{h}\]

\[f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{6g}{h}}\]

24. The number of solution of \(1 - \sin^4 x - 2\cos^4 x = 0\) in the interval \(|0, 2\pi|\) is

(a) 6  
(b) 4  
(c) 2  
(d) 0

Answer (a)

Sol. \(1 - \sin^4 x - 2\cos^4 x = 0\)

\[\Rightarrow 1 - \left(\frac{1 - \cos 2x}{2}\right)^2 - 2\left(\frac{1 + \cos 2x}{2}\right)^2 = 0\]

\[\Rightarrow 3\cos^2 2x + 2\cos 2x - 1 = 0\]

\[\Rightarrow \cos 2x = -1, \frac{1}{3}\]

6 solutions in \([0, 2\pi]\)
25. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere?

(a) Angular momentum
(b) Rotational kinetic energy
(c) Moment of inertia
(d) Angular velocity

Answer (a)

Sol. If radius is increasing keeping mass constant, the mass distribution may change. So moment of inertia will change. But angular momentum will remain constant. To make it constant rotational kinetic energy and angular velocity will change.

\[ \text{kin} = \frac{1}{2} I \omega^2 \]

\[ I \rightarrow \text{moment of inertia} \]

26. If \( n \) is the least positive integer such that

\[ \binom{n}{5} + \binom{n}{5} < \binom{n}{7} \]

the sum of digits of \( n \) is

(a) 6
(b) 5
(c) 4
(d) 3

Answer (c)

Sol. \( \binom{n-1}{5} + \binom{n-1}{5} < \binom{n}{7} \)
\[ \Rightarrow \binom{n-1}{5} < \binom{n}{7} - \binom{n-1}{5} \]
\[ \Rightarrow \binom{n-1}{5} < \binom{n-1}{6} \]
\[ \Rightarrow \frac{(n-1)!}{5!(n-6)!} < \frac{(n-1)!}{6!(n-7)!} \]
\[ \Rightarrow 6 < n - 6 \]
\[ \Rightarrow n > 12 \]
Minimum \( n = 13 \)
Sum = 1 + 3 = 4

27. The flat surface of a solid hemisphere of radius \( r \) is cemented to one flat surface of a cylinder (of identical material) of radius \( r \) and length \( L \). If the total mass is \( M \), moment of inertia of the combination about the axis of the cylinder will be

(a) \( \frac{M}{2} \left[ \frac{L}{15} + 4r \left( \frac{2}{3} \right) \right] \)
(b) \( \frac{M}{3} \left[ \frac{L}{5} + 4r \left( \frac{2}{3} \right) \right] \)
(c) \( \frac{M}{L} \left[ \frac{3}{5} r + \frac{5}{3} \right] \)
(d) \( \frac{M}{L} \left[ \frac{2}{3} \left( \frac{2}{3} \right) \right] \)

Answer (a)

Sol.

\[ M_1 \text{ is mass of hemisphere and } M_2 \text{ is mass of cylinder.} \]
\[ I = \frac{2}{5} M_1 r^2 + \frac{M_2 r^2}{2} \]
\[ M_1 = \frac{2\pi r^3 \rho}{3} \]
\[ M_2 = \pi r^2 L \rho \]
\[ M_1 + M_2 = M \]
\[ \frac{2\pi r^3 \rho + \pi r^2 L \rho}{3} = M \]
\[ \rho = \frac{M}{\pi \left( \frac{2r}{3} + L \right)} \]
\[ I = \frac{2}{5} \frac{2\pi}{3} M \times \frac{1}{\pi \left( \frac{2r}{3} + L \right)} + \frac{\pi r L}{2} \times \frac{M}{\pi \left( \frac{2r}{3} + L \right)} \]
\[ I = \frac{M \left[ \frac{L}{2} + \frac{4r}{15} \right]}{2r/3 + L} \]

28. The limit \( \lim_{x \to \infty} \sqrt{x} + \sqrt{x} - \sqrt{x} \)

(a) does not exist
(b) is \( \frac{1}{2} \)
(c) is 2
(d) is ln 2

Answer (b)

Sol.

\[ \lim_{x \to \infty} \sqrt{x} + \sqrt{x} + \sqrt{x} - \sqrt{x} = \lim_{x \to \infty} \frac{x + \sqrt{x} + \sqrt{x} - x}{\sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x}} \]
\[ = \lim_{x \to \infty} \frac{x + \sqrt{x}}{\sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x}} \]
\[ = \lim_{x \to \infty} \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{1 + \sqrt{x} + \sqrt{x} + \sqrt{x}}} = \frac{1}{2} \]
29. A electron is moving with uniform velocity along a line in the plane of the paper. It is now subjected to a uniform magnetic field \( B \) perpendicular to the paper and going into it. The electron will move in a circular path in the plane of the paper in

(a) Clockwise direction with time period proportional to \( B \)
(b) Anticlockwise direction with time period inversely proportional to \( B \)
(c) Clockwise direction with time period inversely proportional to \( B \)
(d) Anticlockwise direction with time period proportional to \( B \)

Answer (c)

Sol. Magnetic force on negative charge is in opposite direction of \( (\vec{v} \times \vec{B}) \) i.e. clockwise

\[
T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}
\]

\[
T \propto \frac{1}{B}
\]

30. Let \( s_n = 1 + 2 \left( 1 + \frac{1}{n} \right) + 3 \left( 1 + \frac{1}{n} \right)^2 + \ldots + n \left( 1 + \frac{1}{n} \right)^{n-1} \)

Then \( \sum_{n=1}^{\infty} \frac{1}{2^2 \sqrt{s_n}} \) is equal to

(a) \( \frac{4}{3} \)
(b) \( \frac{1}{3} \)
(c) 3
(d) 1

Answer (b)

Sol. Note: We have assumed

\[
s_n = 1 + 2 \left( 1 + \frac{1}{n} \right) + 3 \left( 1 + \frac{1}{n} \right)^2 + \ldots + n \left( 1 + \frac{1}{n} \right)^{n-1}
\]

Let \( x = 1 + \frac{1}{n} \)

\[
s_n = 1 + 2x + 3x^2 + \ldots + nx^{n-1}
\]

\[
xs_n = x + 2x^2 + \ldots + (n-1)x^{n-1} + nx^n
\]

\[
s_n(1-n) = (1 + x + x^2 + \ldots + x^{n-1}) - nx^n
\]

\[
= \frac{1-x^n}{1-x} -nx^n
\]

\[
s_n = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{(1-x)} - nx^n
\]

\[
\frac{1-x^n}{(1-x)^2} - \frac{nx^n}{(1-x)} - nx^n
\]

31. A plane spiral of \( N \) turns, having the radii of internal and external loops as \( r_1 \) and \( r_2 \) carries a current \( I \). The magnetic induction at the centre of the spiral will be

(a) \( \frac{\mu_0 NI}{r_2 - r_1} \ln \frac{r_2}{r_1} \)
(b) \( \frac{\mu_0 NI}{2(r_2 - r_1)} \ln \frac{r_2}{r_1} \)
(c) \( \frac{\mu_0 NI}{r_2 - r_1} \ln \frac{r_2}{r_1} \)
(d) \( \frac{\mu_0 NI}{2(r_2 - r_1)} \ln \frac{r_2}{r_1} \)

Answer (b)

Sol. Number of turn per unit width = \( \frac{N}{r_2 - r_1} \)

"dB" due to small ring element of radius \( r \) and width \( dr \)

\[
dB = \left( \frac{\mu_0}{2} \right) \left( \frac{N}{r_2 - r_1} \right) (dr)x \frac{i}{r}
\]

\[
B_{\text{total}} = \frac{\mu_0 NI}{2(r_2 - r_1)} \int_{r_1}^{r_2} dr = \frac{\mu_0 NI}{2(r_2 - r_1)} \ln \frac{r_2}{r_1}
\]

32. The number of nonzero real solutions of the equations

\[
x^n y = y^2 \quad y^n x = x^{12}
\]

(a) 0
(b) 1
(c) 2
(d) More than 2

Answer (d)
Sol. Taking log on both sides

(i) \((x + y) \log x = 3 \log y\)  
(ii) \((x + y) \log y = 12 \log x\)

(i) and (ii) \(\Rightarrow \frac{(x + y)^2}{12} \log y = 3 \log y\)

\(\Rightarrow \log y = 0 \ or \ (x + y)^2 = 36\)

\(\Rightarrow y = 1 \ or \ x + y = \pm 6\)

\(\Rightarrow x + x^2 = 6 \Rightarrow x = 2, -3\)

\(\therefore \) more than 2 solutions

33. Two identical circular coils are carrying currents \(i_1\) and \(i_2\) are suspended from a torsion free cotton thread in a region of uniform magnetic field \(B\). Each time the coils are given a small angular displacement from their respective equilibrium positions. The time period of the small torsional oscillations were found to be \(T_1\) and \(T_2\). The ratio \(T_1 / T_2\) would be

(a) \(\frac{i_1}{i_2}\)  
(b) \(\frac{i_2}{i_1}\)  
(c) \(\sqrt{i_1/i_2}\)  
(d) \(\sqrt{i_2/i_1}\)

Answer (d)

Sol. As \(T = 2\pi \sqrt{\frac{1}{MB}}\); where \(M = Ni (A_{coil})\)

So, \(T = \frac{1}{\sqrt{i}}\) then \(\frac{T_1}{T_2} = \sqrt{\frac{i_2}{i_1}}\)

34. A triangle has a side of length 8 units, one of the angles of the triangle on this side is 60°. If the inradius of the triangle is \(\sqrt{3}\) units, the perimeter of the triangle is

(a) \(15\sqrt{3}\)  
(b) \(24\)  
(c) \(12\sqrt{3}\)  
(d) \(20\)

Answer (d)

Sol.

\[ \frac{\sqrt{3}}{BP} = \tan 30° \Rightarrow BP = 3\]

\(\Rightarrow BP = BR = 3,\)

\(PC = QC = 8 - 3 = 5\)

Let \(AR = AQ = x\)

\(\therefore \cos 60° = \frac{a^2 + c^2 - b^2}{2ac}\)

\(\Rightarrow \cos 60° = \frac{(3 + x)^2 + (8)^2 - (5 + x)^2}{2 \times 8 \times (x + 3)}\)

\(\Rightarrow x = 2\)

\(\therefore \) Perimeter = 8 + 5 + 3 + 2x

\(= 8 + 5 + 3 + 4\)

\(= 20\)

35. Two cells with emfs \(E_1\) and \(E_2\) have internal resistances \(r_1\) and \(r_2\) respectively. The two cells are connected in series with an external resistance and the current through the external resistance is found to be 1.5 A. When the polarities of the cells are reversed this current is found to be 0.5 A. The ratio of the emfs of the cells is

(a) \(2.5\)  
(b) \(1.5\)  
(c) \(2\)  
(d) \(4\)

Answer (c)

Sol. From Kirchhoff's law

\(E_1 + E_2 = (1.5) (R + r_1 + r_2)\)  
\(\ldots (i)\)

\(E_1 - E_2 = (0.5) (R + r_1 + r_2)\)  
\(\ldots (ii)\)

Now, \(\frac{E_1 + E_2}{E_1 - E_2} = \frac{3 / 2}{1 / 2} = \frac{3}{1}\)

Then, \(E_1 + E_2 = 3E_1 - 3E_2\)

\(4E_2 = 2E_1 \Rightarrow \frac{E_1}{E_2} = 2 : 1\)

36. A points P(8, 4) divides a chord, lying completely in the first quadrant, of a parabola \(y^2 = 4x\) in the ratio 1 : 4. The mid-point of the chord has coordinates

(a) \((17.5, 8)\)
(b) \((18.5, 7)\)
(c) \((19.5, 6)\)
(d) \((20.5, 5)\)
Answer (b)

Sol. Let the two roots be \( p \) and \( p + 1 \).

- Sum of roots = 0 \( \Rightarrow \) third root = \(-2p-1\)
- Given, \( p(p + 1) + p(-2p - 1) + (p + 1)(-2p - 1) = -7 \)
- \( \Rightarrow \) \( p^2 + p - 2 = 0 \Rightarrow p = 1, -2 \)
- Roots = 1, 2, -3 or -2, -1, 3
- Product of roots = -6 or 6
- Therefore, \( \alpha = 6 \) or -6

sum = 6 – 6 = 0.

39. Which of the following physical quantities has the unit volt-second

(a) Energy  
(b) Electric flux  
(c) Magnetic flux  
(d) Inductance

Answer (c)

Sol. \( \text{emf} = \frac{-d\Phi_B}{dt} \)

Therefore, magnetic flux = (EMF) \times \text{time}

unit of magnetic flux = volt \times \text{second}

40. A die is rolled 5 times. The probability that there are at least two equal numbers among the outcomes obtained is

(a) \( \frac{319}{324} \)  
(b) \( \frac{49}{54} \)  
(c) \( \frac{13}{18} \)  
(d) \( \frac{4}{9} \)

Answer (b)

Sol. Required probability

\[ = 1 - \left( \frac{6 \text{C}_5}{5!} \right) \frac{1}{6^5} \]

\[ = \frac{49}{54} \]

41. Imagine a planet of same mass as that of the earth but having a radius twice of that of the earth. A simple pendulum located at some point on its equator failed to show any oscillation when given a small displacement from its equilibrium position. The time taken by this planet to spin once about its own axis is

(a) Nearly 2 hours  
(b) Nearly 4 hours  
(c) Nearly 6 hours  
(d) Nearly 8 hours

Answer (a)

Sol. Let the two roots be \( p \) and \( p + 1 \).

- Sum of roots = 0 \( \Rightarrow \) third root = \(-2p-1\)
- Given, \( p(p + 1) + p(-2p - 1) + (p + 1)(-2p - 1) = -7 \)
- \( \Rightarrow \) \( p^2 + p - 2 = 0 \Rightarrow p = 1, -2 \)
- Roots = 1, 2, -3 or -2, -1, 3
- Product of roots = -6 or 6
- Therefore, \( \alpha = 6 \) or -6

\[ \text{sum} = 6 – 6 = 0. \]
Answer (b)

Sol. On Earth's surface

\[ g = \frac{GM}{R_e^2} \]

On Planet's surface

\[ g' = \frac{GM}{4R_e^2} = \frac{g}{4} \]

As simple pendulum does not oscillate

\[ \therefore g' \text{ on equator is zero} \]

\[ \therefore \frac{g}{4} - 2R_e \omega^2 = 0 \]

\[ \omega^2 = \frac{g}{8R_e} \]

\[ \omega = \sqrt{\frac{g}{8R_e}} \]

\[ 2\pi = \sqrt{\frac{g}{8R_e}} \]

\[ T = 2\pi \sqrt{\frac{8R_e}{g}} \]

\[ T = 2\pi \sqrt{\frac{8 \times 6.4 \times 10^6}{9.8}} \]

\[ T = 3.949 \text{ hours} \]

\[ \therefore T = 4 \text{ hours} \]

43. An alloy of two metals is formed by taking their equal masses and it was found to float on mercury (density 13.6 g cm\(^{-3}\)) with 52.7% above the mercury surface. When an alloy is formed by taking equal volumes of these two metals it was found to float on mercury with 51.5% of its volume below the surface of mercury. The densities of the two metals in g cm\(^{-3}\) are closest to

(a) 6 and 8  
(b) 5 and 9  
(c) 4.5 and 9.5  
(d) 4 and 10

Answer (b)

Sol. When masses are equal

\[ m_1 = m_2 = m \]

\[ 2mg = 0.473 (V_1 + V_2) g \rho_{Hg} \]

\[ 2m = 0.473 \left( \frac{m_1 + m_2}{\rho_1 + \rho_2} \right) \rho_{Hg} \]

\[ 2(\rho_1 \rho_2) = 0.473 (\rho_1 + \rho_2) \rho_{Hg} \]  \hspace{1cm} ...(i)

When volume are equal

\[ V_1 = V_2 = V \]

\[ (m_1 + m_2) g = 0.515 \times 2V \times g \times \rho_{Hg} \]

\[ (\rho_1 + \rho_2) V g = 0.515 \times 2V \times g \times \rho_{Hg} \]

\[ (\rho_1 + \rho_2) = 2 \times 0.515 \times 13.6 \]

\[ = 14.008 = 14 \]  \hspace{1cm} ...(ii)

Putting in equation (i)

\[ 2 \times \rho_1 \rho_2 = 0.473 \times 14 \times 13.6 \]

or, \[ \rho_1 \rho_2 = 45 \]  \hspace{1cm} ...(iii)

Now, \[ (\rho_1 - \rho_2)^2 = (\rho_1 + \rho_2)^2 - 4 \rho_1 \rho_2 \]

\[ = (14)^2 - 4 \times 45 = 196 - 180 \]

\[ = 16 \]

or \[ \rho_1 - \rho_2 = 4 \]  \hspace{1cm} ...(iv)

From eqn (ii) and (iv)

\[ \rho_1 = 9, \rho_2 = 5 \]

44. If \( n \) is the number of function \( f : \{a, b, c, d\} \rightarrow \{a, b, c, d\} \) such that no more than two elements in the domain of \( f \) have the same image, then

(a) \( n \leq 100 \)

(b) \( 100 < n \leq 150 \)

(c) \( 150 < n \leq 200 \)

(d) \( n > 200 \)

Answer (d)
14

**Sol. Case 1**: All elements have different images

\[ n_1 = 4! = 24 \]

**Case 2**: Two elements have same image and two distinct

\[ n_2 = \binom{4}{2} \times 4 \times 3 \times 2 = 144 \]

**Case 3**: Two elements have same image and rest two also have same image but different from the first image.

\[ n_3 = \frac{4!}{2!} \times \binom{4}{2} \times 2 = 36 \]

Total = 204

45. On the rechargeable batteries of 1.5 V often used for digital cameras one can find 2300 mAh or 2800 mAh or something similar is written. This is connected to the

(a) Power that the battery can provide
(b) Current that can be drawn from the battery
(c) Total charge that the battery can supply
(d) Time for which the battery can be used

**Answer (c)**

**Sol.** Rating of mAh represents the charge that battery can supply

mAh \[ \rightarrow \text{Unit of charge. (Current \times time)} \]

for example, 2300 mAh \[ \rightarrow \text{2300 mA for 1 h} \]

or 1150 mA for 2 h

46. The planet in which sun appears to rise in the west is

(a) Venus (b) Uranus
(c) Saturn (d) Mercury

**Answer (a)**

**Sol.** Unlike other planets in the solar system that rotate west to east, Venus spins east to west. Hence sun appears to rise in the west in Venus.

47. Apart from the earth, Aurora phenomena are observed on which of the following planet(s)

(a) Venus (b) Mars
(c) Mercury (d) Jupiter

**Answer (d)**

**Sol.** Aurora occurs on all the planets that have atmosphere and magnetic field. The sun’s charged particles get entrapped in it and emits light due to collision of charged particles with gaseous molecules. Some planets Jupiter, Saturn, Neptune have shown intense auroras.

48. The sum of the last three digits in the expansion of \(5^{2018}\) is

(a) 8 (b) 9
(c) 13 (d) 14

**Answer (c)**

**Sol.**

\[ 5^3 = 125 \]

\[ 5^4 = 125 \times 5 = (100 + 25) \times 5 = 500 + 125 = 625 \]

\[ 5^5 = 625 \times 5 = (600 + 25) \times 5 = 3000 + 125 = 3125 \]

Last 3 digits of \(5^6 = 3125 \times 5 = (3000 + 125 \times 5) = \ldots 625 \)

Last 3 digits of \(5^7 = \ldots 625 \times 5 = (\ldots 600 + 25) \times 5 = \ldots 000 + 125 = \ldots 125 \)

\[ \therefore \text{For even power, last 3 digits is 625 and for odd power 125.} \]

\[ \therefore \text{Last 3 digits of } 5^{1028} = 625 \]

\[ \text{sum} = 6 + 2 + 5 = 13 \]

49. If the wavelength of the incident light changes from 400 nm to 300 nm the stopping potential for photoelectrons emitted from the surface of a material becomes

(a) 0.56 V lower (b) 1.04 V higher
(c) 0.34 V lower (d) 0.56 V higher

**Answer (b)**

**Sol.** Since stopping potential is given as

\[ V = \frac{hc}{\lambda} - \phi \]

Let \(\lambda = 300 \text{ nm}, \text{ then } V = \frac{1}{e} \left( \frac{hc}{300 \text{ nm}} - \phi \right) \]

\[ \lambda = 400 \text{ nm}, \text{ then } V' = \frac{1}{e} \left( \frac{hc}{400 \text{ nm}} - \phi \right) \]

Now,

\[ V - V' = \frac{1}{e} \left( \frac{hc}{300 \text{ nm}} - \frac{hc}{400 \text{ nm}} \right) \]

\[ \Delta V = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 100}{1.6 \times 10^{-19} \times 300 \times 400 \times 10^{-9}} \]

\[ \Delta V = 1.04 \text{ V higher (approximately)} \]

50. Find the integer closest to the integral \( \int_{0}^{6} \left\{ x \right\} dx \), where \( \{x\} \) denotes the largest integer not exceeding \(x\).

(a) 58 (b) 59
(c) 60 (d) 61
52. Let \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b) \) be an ellipse with major axis \( AA' \) and minor axis \( BB' \). Let \( F \) and \( F' \) be the foci of the ellipse, with \( F \) between \( A \) and \( F' \). Suppose \( ABF' \) forms a right-angled triangle. Let \( e \) denote the eccentricity of the ellipse. If \( \phi \) denotes \( \angle FAB \), then \( \tan^2(\phi) \) is equal to

(a) \( \sqrt{e} \)  
(b) \( e \)  
(c) \( e^2 \)  
(d) \( 1 + e \)

Answer (b)

\[ \text{Sol.} \]

\[ Y \]

\[ A \]

\[ (ae, 0) \]

\[ (-ae, 0) \]

\[ B \]

\[ (0, b) \]

\[ B' \]

\[ (-ae, 0) \]

\[ (ae, 0) \]

\[ A(a, 0) \]

\[ X \]

Slope \( AB \) \times \text{slope (BF')} = -1

\[ \Rightarrow \left( \frac{b - 0}{0 - a} \right) \times \left( \frac{b - 0}{0 + ae} \right) = -1 \]

\[ \Rightarrow \frac{b^2}{a^2} = e \quad \text{...(i)} \]

\[ \tan(\pi - \phi) = \frac{b - 0}{0 - a} = \tan \phi = \frac{b}{a} \quad \text{...(ii)} \]

(i) and (ii) \( \Rightarrow \tan^2 \phi = e \)

53. The following graph shows a velocity versus time graph for a ball. Which explanation best fits the motion of the ball as shown by the graph?

(a) The ball falls from a height, is caught, and is thrown down with a greater velocity.

(b) The ball rises to a height, hits the ceiling, and falls down.

(c) The ball falls from a height, hits the floor, and bounces up.

(d) The ball rises to a height, is caught, and then is thrown down with the same velocity.
Taking upward direction positive ball is thrown with speed \(v\). On reaching to the ceiling it reverses its direction and its speed start increasing.

Now, in upward motion \(v_1 = v - gt\)

In downward motion \(v = -v_1 - gt = -(v_1 + gt)\)

\[\therefore\] From given graph best option is (b)

For answering the questions 54 to 57 read the next few lines. Our knowledge of planetary systems is based on the wealth of observations by Copernicus, Tycho Brahe, Johannes Kepler spread over more than a century. Newton may have got some clues about his famous law of universal gravitation, at least the inverse square nature of distance law, from the painstaking work of his predecessors.

54. According to Kepler’s first law, planets go round the Sun in elliptic orbits. If orbit of the earth of eccentricity \(e\) around Sun is divided into two halves by the minor axis, the difference in times spent in the two halves of the orbit is

\[\text{(a) } 2e/\pi \text{ year} \quad \text{(b) } e/\pi \text{ year} \quad \text{(c) } e/(1-e) \text{ year} \quad \text{(d) } 2e^2/(1-e^2) \text{ year}\]

Answer (a)

\[\text{Sol.} \quad (\text{a) } 2e/\pi \text{ year} \quad \text{(b) } e/\pi \text{ year} \quad \text{(c) } e/(1-e) \text{ year} \quad \text{(d) } 2e^2/(1-e^2) \text{ year}\]

55. A planet goes around a star of mass \(M\) and radius \(R\) in an orbit of semi major axis 3R, with the distances as shown. What is the velocity \(V_1\) at the point closest to the star?

\[\text{(a) } (GM/2R)^{1/2} \quad \text{(b) } (2GM/3R)^{1/2} \quad \text{(c) } (4GM/3R)^{1/2} \quad \text{(d) } (GM/6R)^{1/2}\]

Answer (b)

\[\text{Sol.} \quad \text{In elliptical orbit, total energy } E = \frac{-GMm}{2a}\]

\[\text{So,} \quad \frac{1}{2} mv_1^2 = \frac{GMm}{2R} = \frac{GMm}{6R}\]

\[\frac{1}{2} mv_1^2 = \frac{GMm}{6R} + \frac{GMm}{2R}\]

\[\frac{m}{2} v_1^2 = \left(\frac{-1 + \frac{3}{6}}{6}\right) \frac{GMm}{R}\]

\[v_1^2 = \frac{2GM}{3R}\]

\[v_1 = \sqrt{\frac{2GM}{3R}}\]

56. What are the eccentricity and length of semi minor axis in the orbit in Q.55?

\[\text{(a) } 0.30, 2.50R \quad \text{(b) } 0.33, 2.00R \quad \text{(c) } 0.33, 2.83R \quad \text{(d) } 0.25, 2.75R\]
17

**Answer (c)**

**Sol.** From diagram,

\[
\text{ae} = R
\]

or, \(e = \frac{R}{3R} = \frac{1}{3}\)

\[
e = \sqrt{1 - \frac{b^2}{a^2}}
\]

\[
\frac{1}{9} = 1 - \frac{b^2}{a^2}
\]

\[
\frac{b^2}{a^2} = \frac{8}{9}
\]

\[
b = a\sqrt{\frac{8}{9}} = 3R\sqrt{\frac{8}{9}} = 2.83R
\]

57. If the earth of mass M is assumed to be a sphere of 6400 Km, with what velocity must a projectile be fired from the earth's surface in order that its subsequent path may be an ellipse with major axis 80,000 Km? [Take the product \(GM = 4.0 \times 10^{14} \text{ m}^3\text{s}^{-2}\)]

(a) 10.70 Km/s  
(b) 11.20 Km/s  
(c) 9.50 Km/s  
(d) 11.70 Km/s

**Answer (a)**

**Sol.**

\[
V^2 = 2GM\left[\frac{1}{R_E} - \frac{2}{25R_E}\right]
\]

\[
V^2 = \frac{2GM}{R_E} \left[\frac{23}{25}\right]
\]

\[
V = \sqrt{\frac{23}{25}} \sqrt{\frac{2GM}{R_E}} = \sqrt{\frac{23}{25}} \times \frac{2 \times 4 \times 10^{14}}{6400 \times 10^3}
\]

\[\Rightarrow V = 10.70 \text{ Km/s}\]

58. Consider the cubic curve \(y = 2x^3 - 12x^2 + 18x + 5\). Let A and C be its extremum points. The tangents at A and C to the curve intersect it again at two other points B and D respectively. The area of the quadrilateral ABCD is

(a) 12  
(b) 24  
(c) 36  
(d) 48

**Answer (b)**

**Sol.**

\[
y = 2x^3 - 12x^2 + 18x + 5
\]

\[
y' = 6x^2 - 24x + 18 \Rightarrow y' = 0
\]

\[\Rightarrow x = 1 \text{ or } x = 3\]

Now tangent at A and C cut the graph at B and D respectively.

So put \(y = 13\) for getting point B and put \(y = 5\) for getting point D.

So point B(4, 13) and D(0, 5).

Now area of parallelogram ABCD is

\[
\frac{1}{2} \times 3 \times 8 + \frac{1}{2} \times 3 \times 8 = 24 \text{ sq units}
\]

59. A crater on the surface of the moon has a diameter of 80 km. If the distance to earth and moon is \(3.78 \times 10^5 \text{ km}\) then the visual angle in degree is

(a) 0.012  
(b) 0.021  
(c) 0.019  
(d) 0.026

**Answer (a)**
61. There is a uniformly charged non-conducting solid sphere made of material of dielectric constant 1. If the electric potential at infinity is taken to be zero, then the potential at its surface is \( V \). If we take the electric potential at its surface to be zero, then the potential at the centre will be

(a) \( \frac{3V}{2} \)  
(b) \( \frac{V}{2} \)  
(c) \( V \)  
(d) Zero

Answer (b)

Sol. In general potential at centre, \( V_c = \frac{3V}{2} \)

\[ V = 0 \]

\[ V_c - V_s = \frac{V}{2} \]

62. Suppose \( 5 \cos x + 12 \cos y = 13 \). The maximum possible value of \( 5 \sin x + 12 \sin y \) is

(a) 13  
(b) \( 120 \)  
(c) \( 240 \)  
(d) 13

Answer (b)

Sol. \( (5 \cos x + 12 \cos y)^2 = 169 \)

\[ 25(1 - \sin^2 x) + 144(1 - \sin^2 y) + 120 \cos x \cos y = 169 \]

\( (5 \sin x + 12 \sin y)^2 = 120 \cos(x - y) \)

\( (5 \sin x + 12 \sin y)^2 \) max = \( \sqrt{120} \)

63. If speed of light (\( C \)), acceleration due to gravity (\( g \)) and pressure (\( P \)) are taken to be fundamental units, then dimension of universal gravitational constant (\( G \)) is

(a) \( C g P^{-3} \)  
(b) \( C^2 g^3 P^{-2} \)  
(c) \( C^0 g^2 P^{-1} \)  
(d) \( C^2 g^2 P^{-2} \)

Answer (c)

Sol. \([C] = [L^1 T^{-1}]\)

\([g] = [L^1 T^{-2}]\)

\([P] = [M^1 L^{-1} T^{-2}]\)

\([G] = [M^{-1} L^3 T^{-2}]\)

\([G] = [C]^4 [g]^6 [P]^1\]

solving we get \( \gamma = -1 \)
64. Let \( f(x) = \begin{cases} \frac{\pi}{2} \sin x, & \text{for } 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \leq x \leq \pi \end{cases} \). Then

(a) Nowhere continuous \((0, \pi)\)

(b) Continuous on \((0, \pi)\) except at \(x = \frac{\pi}{2}\)

(c) Continuous on \((0, \pi)\), but nowhere differentiable

(d) Differentiable at all points of \((0, \pi)\)

Answer (d)

Sol. 

\[
\begin{align*}
\text{Let } f(x) &= \begin{cases} 
\frac{\pi}{2} \sin x, & 0 < x < \frac{\pi}{2} \\
\frac{\pi}{2}, & \frac{\pi}{2} \leq x < \pi
\end{cases} \\
\Rightarrow f(x) &= \frac{\pi}{2} \\
\therefore f(x) &= \frac{\pi}{2}
\end{align*}
\]

\begin{align*}
\text{f} \left( \frac{\pi}{2}^- \right) &= \frac{\pi}{2} \\
\text{f} \left( \frac{\pi}{2}^+ \right) &= \frac{\pi}{2}
\end{align*}

\begin{align*}
\Rightarrow f(x) &= \text{continuous at } x = \frac{\pi}{2} \\
\therefore f(x) &= \text{continuous in } (0, \pi)
\end{align*}

\begin{align*}
\text{f}' \left( \frac{\pi}{2}^- \right) &= \pi \cos x \bigg|_{x = \frac{\pi}{2}} = 0 \\
\text{f}' \left( \frac{\pi}{2}^+ \right) &= \pi \cos x \bigg|_{x = \frac{\pi}{2}} = 0
\end{align*}

\begin{align*}
\therefore f(x) &= \text{differentiable at } x = \frac{\pi}{2}
\end{align*}

\begin{align*}
in \left( 0, \frac{\pi}{2} \right), y = \frac{\pi}{2} \sin x \text{ is differentiable}
\end{align*}

\begin{align*}
in \left( \frac{\pi}{2}, \pi \right), y = \frac{\pi}{2} \text{ is differentiable}
\end{align*}

\begin{align*}
\therefore f(x) &= \text{differentiable at all points of } (0, \pi)
\end{align*}

65. A wave propagating along X-axis is represented by \( y = a \sin(At - Bx + C) \) where \( y \) is the displacement of the particle, \( A \) the amplitude of the wave and \( t \) is the time. If \( A, B \) and \( C \) are three constants then the dimension of \( \frac{aBC}{A} \) is the same as that of

(a) Length
(b) Mass
(c) Time
(d) Velocity

Answer (d)

Sol. 

\[
\begin{align*}
\text{Answer (c)} \\
\text{Sol. } [a] &= L^1 \\
A &= T^{-1} \\
B &= L^{-1} \\
C &= L^0 T^0 \\
[\frac{abc}{A}] &= [L^1 T^{-1}] = [T]
\end{align*}
\]

66. The sides of a triangle are 8, 10, \( x \) where \( x \) is a positive integer. The number of possible values of \( x \) for which triangle becomes acute is

(a) 6 
(b) 5 
(c) 4 
(d) 3

Answer (a)

Sol. 

\[
\begin{align*}
\cos A &= \frac{10^2 + 8^2 - x^2}{2 \times 10 \times 8} > 0 \\
\Rightarrow x^2 &< 10^2 + 8^2 \\
\Rightarrow x^2 &> 10^2 - 8^2 \\
\Rightarrow x &> 6 \text{ or } x < 13 \\
\Rightarrow x &= 7, 8, 9, 10, 11, 12 \\
\text{Total number of solutions} &= 6
\end{align*}
\]

67. The speed \( v \) in m/s and time \( t \) in second for an object moving along a straight line are related as \( t^2 - 2\sqrt{2} vt + 50 = 0 \). The possible value of \( v \) is

(a) \( v \geq 5 \text{ m/s only} \)
(b) \( v \geq 10 \text{ m/s only} \)
(c) \( v \geq 15 \text{ m/s only} \)
(d) \( v \geq 25 \text{ m/s only} \)

Answer (a)

Sol. 

\[
\begin{align*}
D &= 0 \\
8v^2 - 200 &\geq 0 \\
v &\geq 5 \text{ m/s} \\
\therefore v_{\text{min}} &= 5 \text{ m/s}
\end{align*}
\]
68. There are n teachers in a school and all possible 4 member committees are formed. Among these, exactly $\frac{1}{20}$ part of the committees have 2 fixed members. The sum of the digits of n is

(a) 8 (b) 7 (c) 6 (d) 5

Answer (b)

Sol. Total number of committees = $nC_4$

Number of committees which have 2 fixed member = $n–2C_2$

Therefore,

$$\frac{1}{20} nC_4 = n–2C_2$$

$$\Rightarrow \frac{n(n–1)(n–2)(n–3)}{4 \times 3 \times 2 \times 1} = 20 \times \frac{(n–2)(n–3)}{2 \times 1}$$

$$\Rightarrow n(n–1) = 240 \Rightarrow (n + 15)(n – 16) = 0$$

$$\Rightarrow n = 16$$

Sum of digits = 1 + 6 = 7

69. A chamber is enclosed in a thermally insulated cover and a partition wall separates it into two parts A and B. Part A is filled up with an ideal gas at pressure $p_A$ and as a volume $V_A$. The other part (part B) is evacuated and has a volume $V_B$. Assume this part to be vacuum. The partition wall is now removed. When the equilibrium is set in. The pressure $p$ in the entire chamber is

(a) $p = p_A$ (b) $p = p_A \frac{V_A + V_B}{V_B}$ (c) $p = \frac{p_A V_A}{V_A + V_B}$ (d) $p = \frac{p_A V_B}{V_A + V_B}$

Answer (c)

Sol. As it expands against vacuum

$\Delta u = 0$

$\Delta w = 0$

$T = $ constant

$p_A V_A = p(V_A + V_B)$

$p = \frac{p_A V_A}{V_A + V_B}$

70. Let $(1 + x – 3x^2)^{2018} = a_0 + a_1 x + a_2 x^2 + ... + a_{4036} x^{4036}$. The last digit of $a_0 + a_2 + a_4 + ... + a_{4036}$ is

(a) 0 (b) 5 (c) 7 (d) 9

Answer (b)

Sol. $(1 + x – 3x^2)^{2018} = a_0 + a_1 x + a_2 x^2 + ... + a_{4036} x^{4036}$ ... (1)

Put $x = 1$ and $x = -1$ in (1) to get

$$1 = a_0 + a_1 + a_2 + ... + a_{4036} \quad \text{...(2)}$$

and $3^{2018} = a_0 + a_1 + a_2 + a_3 + ... + a_{4036} \quad \text{...(3)}$

$(2) + (3)$

$$\Rightarrow a_0 + a_2 + a_4 + ... + a_{4036} = \frac{1}{2}(3^{2018} + 1)$$

$$= \frac{1}{2}(10^{1009} + 1)$$

$$= \frac{1}{2}[10^{1009}C_0 10^{1009} - 10^{1009}C_1 10^{1008} + ... + 1009 \times 10]$$

$$= \frac{1}{2}[10^{1009}C_0 (10)^{1009} - 10^{1009}C_1 (10)^{1008}$$

$$+ ... - 1009C_{1007} \times 10^2] + 5045$$

last digit = 5

71. The figure shows some of the field lines of an electric field. The figure suggests that

(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$ (c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

Answer (c)

Sol. Degree of closeness of field lines is proportional to field strength

$\therefore E_A = E_C > E_B$

72. The value of the integral $\int_{0}^{2} x \cos(\pi x)dx$, where $\{x\}$ denotes the fractional part of $x$, is

(a) 0 (b) $\frac{4}{\pi^2}$ (c) $-\frac{4}{\pi^2}$ (d) $-\frac{2}{\pi^2}$
Answer (c)

Sol. \[ I = \int_0^1 x \cos(\pi x) \, dx = \int_0^2 x \cos(\pi(x-[x])) \, dx \]

\[ = \int_0^1 x \cos(\pi x - 0) \, dx + \int_1^2 x \cos(\pi x - 1) \, dx \]

\[ = \int_0^1 x \cos \pi x \, dx + \int_1^2 -x \cos \pi x \, dx \]

\[ l_x = \int_0^1 x \cos \pi x \, dx \]

\[ = x - \sin \frac{\pi x}{\pi} \bigg|_0^1 \]

\[ = \left[ x \sin \frac{\pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right]_0^1 \]

\[ \therefore I = \frac{-4}{\pi^2} \]

73. The moment of the force \( F = 4i + 5j - 6k \) acting at the point \( (2, 0, -3) \) and about the axis passing through a point \( (2, -2, -2) \) is given by

(a) \(-7i - 4j - 8k\)
(b) \(-7i - 8j - 4k\)
(c) \(-4i - j - 8k\)
(d) \(-8i - 4j - 7k\)

Answer (a)

Sol. \( \vec{r} = (2 - 2)\hat{i} + (0 + 2)\hat{j} + (-3 + 2)\hat{k} \)

\[ = \hat{0}i + 2j - \hat{k} \]

\[ \vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k} \]

\[ \therefore \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 4 & 5 & -6 \end{vmatrix} \]

\[ = \hat{i}(12 + 5) + \hat{j}(-4 - 0) + \hat{k}(0 - 8) \]

\[ \therefore \text{moment of force} = -7\hat{i} - 4\hat{j} - 8\hat{k} \]

74. If \( \alpha, \beta, \gamma \) are the roots of \( x^2 + x + 2 = 0 \), then \( \frac{\alpha^4 + \beta^4 + \gamma^4}{\alpha^2 + \beta^2 + \gamma^2} \) equals

(a) \( \frac{1}{7} \)
(b) 7
(c) \( \frac{1}{6} \)
(d) 6

Answer (b)

Sol. \[
\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = 0 \Rightarrow (x + 3) x - 2 - x = 0 \\
\Rightarrow (x + 3) (x - 2) (x - 1) = 0 \]

\[ \Rightarrow x = 1, 2, -3 \]

\[ \alpha = 1, \beta = 2, \gamma = -3 \]

\[ \frac{\alpha^4 + \beta^4 + \gamma^4}{\alpha^2 + \beta^2 + \gamma^2} = \frac{1 + 16 + 81}{1 + 4 + 9} = 7 \]

75. If all nuclear reactions in the sun now were to suddenly stop for ever, then

(a) Distances between planets and sun would decrease.
(b) Angular momentum of planets would increase.
(c) Inner planets will be engulfed by the sun.
(d) Speed of rotation of the sun about its own axis would increase.

Answer (d)

Sol. When nuclear reactions of sun stop, its radius will decrease, and therefore the angular speed of rotation about axis will increase.
76. Three well known stars (a) Procyon (b) Antares and (c) Vega are respectively in the constellation
(a) Orion, Sagittarius and Scorpios
(b) Orion, Taurus and Ursa major
(c) Canis minor, Scorpius and Lyra
(d) Scorpius, Canes minor and Leo
Answer (c)
Sol. Procyon – Canis
Antares – Scorpius
Vega – Lyra
\( \therefore \) Option (c)

77. One gram of Radium, with atomic weight 226, emits \( 4 \times 10^{10} \) particles per second. The half-life of Radium is
(a) \( 4.6 \times 10^{10} \) s
(b) \( 4.6 \times 10^9 \) s
(c) \( 4.6 \times 10^{12} \) s
(d) \( 4.6 \times 10^{14} \) s
Answer (a)
Sol.
\[ \frac{dN}{dt} = \lambda N \]
\[ \Rightarrow 4 \times 10^{10} = \frac{6.02 \times 10^{23}}{226} \]
\[ \therefore t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693 \times 6.02 \times 10^{23}}{226 \times 4 \times 10^{10}} \]
\[ = 4.6 \times 10^{10} \text{ s} \]

78. Let \( \langle a_n \rangle \geq 0 \) be a geometric progression with common
ratio \( r \), \( |r| < 1 \). Let \( s_1 = \sum_{k=0}^{\infty} a_k \), \( s_2 = \sum_{k=0}^{\infty} a_{2k} \) and
\[ s_3 = \sum_{k=0}^{\infty} a_{3k} \]. Suppose \( \frac{s_1}{s_2} = \frac{5}{4} \). Then \( \frac{s_2}{s_3} \) equals
(a) \( \frac{5}{4} \)
(b) \( \frac{25}{24} \)
(c) \( \frac{21}{20} \)
(d) \( \frac{9}{10} \)
Answer (c)
Sol.
\[ s_1 = \frac{a_0}{1-r}, \quad s_2 = \frac{a_0}{1-r^2}, \quad s_3 = \frac{a_0}{1-r^3} \]
\[ \Rightarrow \frac{s_1}{s_2} = \frac{5}{4} \Rightarrow \frac{1-r^2}{1-r} = \frac{5}{4} \Rightarrow 1+r = \frac{5}{4} \Rightarrow r = \frac{1}{4} \]

79. An electric dipole of moment \( p \) is lying on a plane in a uniform electric field \( E_0 \) with the dipole axis along the field. The dipole on the plane is rotated by an angle 60º keeping its centre of mass fixed. The potential energy of the dipole in its new position will be
(a) \(-pE_0\)
(b) \(-\frac{pE_0}{2}\)
(c) \(-\frac{pE_0}{3}\)
(d) \(-\frac{pE_0}{4}\)
Answer (b)
Sol.
\[ U = -p \cdot E \cos 60^\circ \]
\[ = -\frac{pE}{2} \]

80. Let \( I_1 = \int_0^1 \frac{dx}{1+\sqrt[3]{x}} \) and \( I_2 = \int_0^1 \frac{dx}{1+\sqrt[4]{x}} \). Then \( 4I_1 + 3I_2 \) equals
(a) 3
(b) 4
(c) 6
(d) 7
Answer (b)
Sol.
\[ I_1 = \int_0^1 \frac{dx}{1+\sqrt[3]{x}}, \quad \text{Put } x = t^3 \]
\[ I_2 = \int_0^1 \frac{dx}{1+\sqrt[4]{x}}, \quad \text{Put } x = t^4 \]
\[ 4I_1 + 3I_2 = 12 \int_0^1 \frac{t^2 + t^3}{1+t} \, dt = 12 \int_0^1 t^2 \, dt = 12 \frac{1}{3} = 4 \]