

MATHEMATICS

**SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The 3 digit numbers which are divisible either 3 or 4 but not divisible by 48  
 (1) 414 (2) 420  
 (3) 429 (4) 432

**Answer (4)**

**Sol.** No's divisible by 3 = 300  
 No's divisible by 4 = 225  
 No's divisible by 12 = 75  
 No's divisible by 48 = 18  
 Total no's = 300 + 225 - 75 - 18  
 = 432

2. The letters of the word GHOTU is arranged alphabetically as in a dictionary. The rank of the word TOUGH is  
 (1) 84 (2) 79  
 (3) 74 (4) 89

**Answer (4)**

**Sol.** GHOTU

G \_ \_ \_ \_ 24  
 H \_ \_ \_ \_ 24  
 O \_ \_ \_ \_ 24  
 T G \_ \_ \_ 6  
 T H \_ \_ \_ 6  
 T O G \_ \_ 2  
 T O H \_ \_ 2  
 T O U G H 1  
 = 89

3.  $\int_{1/2}^2 \frac{\tan^{-1} x}{x} dx$  equals

- (1)  $\frac{\pi}{2} \ln 2$  (2)  $\frac{\pi}{4} \ln 2$   
 (3)  $\pi \ln 2$  (4)  $\ln 2$

**Answer (1)**

**Sol.**  $I = \int_{1/2}^2 \frac{\tan^{-1} x}{x} dx$

Put  $x = \frac{1}{t}$

$$I = \int_{1/2}^2 \frac{\tan^{-1} \left(\frac{1}{t}\right)}{t} dt$$

$$\therefore 2I = \int_{1/2}^2 \frac{\pi}{2} \frac{dt}{t}$$

$$\Rightarrow I = \frac{\pi}{2} \ln 2$$

4. Shortest distance between lines:

$$\frac{x-1}{2} = \frac{2y-2}{3} = \frac{z-3}{1} \text{ and } \frac{x-2}{3} = \frac{y-1}{2} = \frac{z+2}{4} \text{ is}$$

- (1)  $\frac{13}{\sqrt{165}}$  (2)  $\frac{15}{\sqrt{165}}$   
 (3)  $\frac{18}{\sqrt{165}}$  (4)  $\frac{19}{\sqrt{165}}$

**Answer (1)**

**Sol.** Lines are

$$\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-3}{1} \text{ \& } \frac{x-2}{3} = \frac{y-1}{2} = \frac{z+2}{4}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 2 & 4 \end{vmatrix} = \hat{i}(4) - \hat{j}(5) + \hat{k}\left(-\frac{1}{2}\right)$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{i} - 5\hat{k}) \cdot \left(4\hat{i} - 5\hat{j} - \frac{\hat{k}}{2}\right)$$

$$= 4 + \frac{5}{2} = \frac{13}{2}$$

$$\therefore \text{Shortest distance} = \frac{\frac{13}{2}}{\sqrt{16 + 25 + \frac{1}{4}}} = \frac{3}{\sqrt{165}}$$

5.  $R = \{(a, b) : 2a + 3b \text{ is divisible by } 5 \text{ and } a, b \in \mathbb{N}\}$  is
- (1) Transitive but not symmetric
  - (2) Equivalence relation
  - (3) Symmetric but not transitive
  - (4) Not equivalence

**Answer (2)**

**Sol.** Let  $(a, b) \in R$

$$f(a, b) = 2a + 3b$$

For Reflexive

$$f(a, a) = 2a + 3a = 5a \text{ i.e., divisible by } 5$$

$$\Rightarrow (a, a) \in R$$

For symmetric

$$f(b, a) = 2b + 3a = \underbrace{5a}_{\text{divisible by } 5} + \underbrace{5b}_{\text{divisible by } 5} - (2a + 3b)$$

$f(b, a)$  is divisible by 5

$$\Rightarrow (b, a) \in R$$

For transitive

$$f(a, b) = 2a + 3b \text{ is divisible by } 5$$

$$f(b, c) = 2b + 3c \text{ is divisible by } 5$$

$$\Rightarrow 2a + 5b + 3c \text{ is divisible by } 5$$

So,  $2a + 3c$  is divisible by 5

$$\Rightarrow (a, c) \in R$$

6.  $(\sim A) \vee B$  is equivalent to

- (1)  $A \rightarrow B$
- (2)  $A \leftrightarrow B$
- (3)  $\sim A \wedge B$
- (4)  $B \rightarrow A$

**Answer (1)**

**Sol.** Making truth table

A	B	$\sim B$	$(\sim A) \vee B$	$A \rightarrow B$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The truth table clearly shows.

$$(\sim A) \vee B \equiv A \rightarrow B$$

7. The value of  $\int_{\frac{1}{2}}^2 \left( \frac{t^4 + 1}{t^6 + 1} \right) dt =$

- (1)  $\tan^{-1} 2 + \tan^{-1} 8 + \frac{2\pi}{3}$
- (2)  $2\tan^{-1} 2 + \frac{2}{3}\tan^{-1} 8 - \frac{2\pi}{3}$
- (3)  $2\tan^{-1} 2 + \frac{2}{3}\tan^{-1} 8 + \frac{2\pi}{3}$
- (4)  $2\tan^{-1} 2 - \frac{2}{3}\tan^{-1}(8) + \frac{2\pi}{3}$

**Answer (2)**

**Sol.** 
$$\int_{\frac{1}{2}}^2 \frac{t^4 + 1}{t^6 + 1} dt$$

$$= \int_{\frac{1}{2}}^2 \frac{(t^4 + 1)(t^2 + 1)}{(t^6 + 1)(t^2 + 1)} dt = \int_{\frac{1}{2}}^2 \frac{t^6 + 1 + t^2(t^2 + 1)}{(t^6 + 1)(t^2 + 1)} dt$$

$$= \int_{\frac{1}{2}}^2 \frac{dt}{t^2 + 1} + \frac{1}{3} \int_{\frac{1}{2}}^2 \frac{3t^2 dt}{t^6 + 1}$$

$$= \tan^{-1} t \Big|_{\frac{1}{2}}^2 + \frac{1}{3} \tan^{-1}(t^3) \Big|_{\frac{1}{2}}^2$$

$$= \left( \tan^{-1} 2 - \tan^{-1} \left( \frac{1}{2} \right) \right) + \frac{1}{3} \left( \tan^{-1}(8) - \tan^{-1} \left( \frac{1}{8} \right) \right)$$

$$= \left( \tan^{-1} 2 - \cot^{-1} 2 \right) + \frac{1}{3} \left( \tan^{-1} 8 - \cot^{-1} 8 \right)$$

$$= \tan^{-1} 2 - \left( \frac{\pi}{2} - \tan^{-1} 2 \right) + \frac{1}{3} \left( \tan^{-1} 8 - \left( \frac{\pi}{2} - \tan^{-1} 8 \right) \right)$$

$$= 2 \left( \tan^{-1} 2 \right) + \frac{2}{3} \tan^{-1} 8 - \frac{\pi}{2} - \frac{\pi}{6}$$

$$= 2 \tan^{-1} 2 + \frac{2}{3} \tan^{-1} 8 - \frac{2\pi}{3}$$

8. Area of region

$$|\cos x - \sin x| \leq y \leq \sin x \text{ for } x \in (0, \pi/2) \text{ is}$$

- (1)  $(-1 + 2\sqrt{2})$  sq. unit
- (2)  $\left( 1 - \frac{1}{\sqrt{2}} \right)$  sq. unit
- (3)  $(\sqrt{5} + 1 - 2\sqrt{2})$  sq. unit
- (4)  $(\sqrt{5} - \sqrt{2})$  sq. unit

**Answer (3)**

Sol.  $A = \int_0^{\pi/2} (\sin x - |\cos x - \sin x|) dx$

Where  $\theta = \tan^{-1} \frac{1}{2}$

$= \int_0^{\pi/4} (\sin x - \cos x + \sin x) dx +$

$\int_{\pi/4}^{\pi/2} (\sin x + \cos x - \sin x) dx$

$= -2\cos x - \sin x \Big|_0^{\pi/4} + \left(1 - \frac{1}{\sqrt{2}}\right)$

$= -\left(\sqrt{2} + \frac{1}{\sqrt{2}} - 2\cos\theta - \sin\theta\right) + 1 - \frac{1}{\sqrt{2}}$

$= -\sqrt{2} - \frac{1}{\sqrt{2}} + (2\cos\theta + \sin\theta) + 1 - \frac{1}{\sqrt{2}}$

$= 1 - 2\sqrt{2} + 2 \cdot \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}}$

$= \sqrt{5} + 1 - 2\sqrt{2}$

9. For solution of differential equation

$x \ln x \frac{dy}{dx} + y = x^2 \ln x, y(2) = 2$  then  $y(e)$  is equal to

(1)  $1 + \frac{e^2}{4}$

(2)  $1 - \frac{e^2}{4}$

(3)  $\frac{e^2}{2}$

(4)  $1 + \frac{e^2}{2}$

Answer (1)

Sol.  $x \ln x \frac{dy}{dx} + y = x^2 \ln x$

$\frac{dy}{dx} + \frac{y}{x \ln x} = x$

I.F =  $e^{\int \frac{1}{x \ln x} dx} = e^{\ln|\ln x|} = |\ln x|$

Solution of equation is

$y(IF) = \int x \cdot |\ln x| dx$

$y |\ln x| = |\ln x| \left(\frac{x^2}{2} - \frac{x^2}{4} + c\right)$

$x = 2, 2 |\ln 2| = |\ln 2| \cdot 2 - 1 + c$

$\Rightarrow c = 1$

For  $x = e$

$y = \frac{e^2}{2} - \frac{e^2}{4} + 1$

$\Rightarrow y(e) = 1 + \frac{e^2}{4}$

10. Let  $f(x) = x^2 + 2x + 5$  and  $\alpha, \beta$  be roots of  $f\left(\frac{1}{t}\right) = 0$ ,

then  $\alpha + \beta =$

(1)  $\frac{-2}{5}$

(2)  $-2$

(3)  $\frac{5}{2}$

(4)  $\frac{-5}{2}$

Answer (1)

Sol.  $f(x) = x^2 + 2x + 5$

$f\left(\frac{1}{t}\right) = 0 \Rightarrow \frac{1}{t^2} + \frac{2}{t} + 5 = 0$

$\Rightarrow 5t^2 + 2t + 1 = 0 \quad [t \neq 0]$

This equation has roots  $\alpha$  and  $\beta$ .

$\therefore \alpha + \beta = \frac{-2}{5}$

11. If the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{1}$  and

$\frac{x-11}{4} = \frac{y-9}{2} = \frac{z-4}{3}$  intersects at point  $p$ , then

perpendicular distance of  $p$  from plane  $3x + 2y + 6z = 10$  is

(1)  $\frac{2}{7}$

(2)  $\frac{3}{7}$

(3)  $\frac{4}{7}$

(4)  $\frac{5}{7}$

Answer (2)

Sol. Given

$L_1 \equiv \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{1} = \lambda$

and  $L_2 \equiv \frac{x-11}{4} = \frac{y-9}{2} = \frac{z-4}{3} = \mu$

Finding intersection, we get  $\lambda = 1, \mu = -2$

$\therefore p \equiv (3, 5, -2)$

Distance from given plane =  $\left| \frac{9+10-12-10}{\sqrt{9+4+36}} \right| = \frac{3}{7}$

12. If  $\cos^2 2x - \sin^4 x - 2\cos^2 x = \lambda$ , has a solution  $\forall x \in R$ , then the range of  $\lambda$  is

(1)  $\left[-\frac{1}{2}, 1\right]$

(2)  $\left[-\frac{4}{3}, 0\right]$

(3)  $(0, 2)$

(4) None of these

Answer (2)

**Sol.**  $\cos^2 2x - \sin^4 x - 2\cos^2 x = \lambda$

$$\Rightarrow (\cos^2 x - \sin^2 x)^2 - \sin^4 x - 2\cos^2 x = \lambda$$

$$\Rightarrow 3\cos^4 x - 4\cos^2 x = \lambda$$

$$\Rightarrow 3\left(\cos^2 x - \frac{2}{3}\right)^2 - \frac{4}{9} = \lambda$$

$$\lambda_{\min} = -\frac{4}{3}$$

$$\lambda_{\max} = 0$$

$$\therefore \lambda \in \left[-\frac{4}{3}, 0\right]$$

13.  $\vec{a} = 9\hat{i} + 2\hat{k}$ ,  $\vec{b} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{c} = 7\hat{i} - 3\hat{j} + 2\hat{k}$  are three given vectors. Let there be a  $\vec{r}$  such that  $(\vec{r} \times \vec{b}) + (\vec{b} \times \vec{c}) = \vec{0}$  and  $\vec{r} \cdot \vec{a} = 0$  then  $\vec{r} \cdot \vec{c}$  is

(1)  $\frac{280}{11}$  (2) 28

(3)  $\frac{279}{13}$  (4)  $\frac{290}{11}$

**Answer (1)**

**Sol.**  $\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$$(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$$

$$\Rightarrow \vec{r} = \vec{c} + \lambda \vec{b}$$

$$\vec{r} \cdot \vec{a} = 0$$

$$\vec{c} \cdot \vec{a} + \lambda \vec{b} \cdot \vec{a} = 0$$

$$67 + \lambda(11) = 0$$

$$\lambda = -\frac{67}{11}$$

$$\vec{r} \cdot \vec{c} = (\vec{c} + \lambda \vec{b}) \cdot \vec{c}$$

$$= |\vec{c}|^2 + \lambda \vec{b} \cdot \vec{c}$$

$$= 62 - \frac{67}{11}(7 - 3 + 2)$$

$$= 62 - \frac{67}{11}(6)$$

$$= \frac{682 - 402}{11} = \frac{280}{11}$$

14. For observation set x data obtained is

$$x_i = \{11, 12, 13, \dots, 41\}$$

For another observation set y data obtained is

$$y_i = \{61, 62, 63, \dots, 91\}$$

Then value of  $\bar{x} + \bar{y} + \sigma^2$  where  $\bar{x}, \bar{y}$  are means of respective data set while  $6^2$  is variance of combined data is

(1) 801 (2) 754

(3) 807 (4) 774

**Answer (3)**

**Sol.**  $\bar{x} = \frac{\frac{31}{2}(11+41)}{31} = \frac{1}{2} \times 52 = 26$

$$\bar{y} = \frac{\frac{31}{2}(61+91)}{31} = \frac{1}{2} \times 152 = 76$$

$$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{62} - \left(\frac{\sum x_i + \sum y_i}{62}\right)^2$$

$$= \frac{(11^2 + 12^2 + \dots + 41^2) + (61^2 + 62^2 + \dots + 91^2)}{62} - (51)^2$$

$$= \left(\frac{41 \cdot 42 \cdot 83}{6} + \frac{10 \cdot 11 \cdot 21}{6}\right) + \left(\frac{91 \cdot 92 \cdot 183}{6} - \frac{60 \cdot 61 \cdot 121}{6}\right) - (51)^2$$

$$= \frac{(41 \cdot 7 \cdot 83 - 11 \cdot 35) + (91 \cdot 46 \cdot 61 - 10 \cdot 61 \cdot 121)}{62} - (51)^2$$

$$= \frac{23436 + 181536}{62} - (51)^2$$

$$= 3306 - 2601 = 705$$

$$\bar{x} + \bar{y} + \sigma^2$$

$$= 26 + 76 + 705$$

$$= 807$$

15. If curve represented by  $y = \frac{(x-a)}{(x-3)(x-2)}$  passes

through (1, -3) then equation of normal at (1, -3) to the curve is given by

(1)  $2x + 3y = -7$  (2)  $3x - 2y = 9$

(3)  $3x - 4y = 21$  (4)  $x - 4y = 13$

**Answer (4)**

Sol. If passes through (1, -3)

$$-3 = \frac{1-a}{(-2)(-1)}$$

$$\boxed{a=7}$$

$$f'(x) = \frac{(x-3)(x-2) - (x-7)(2x-5)}{((x-3)(x-2))^2}$$

$$f'(1) = \frac{2-18}{(2)^2} = -4$$

Slope of normal =  $\frac{1}{4}$

Equation of normal

$$y+3 = \frac{1}{4}(x-1)$$

or  $4y+12 = x-1$

or  $x-4y = 13$

16.

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the number of four-digit numbers  $N$  such that  $GCD(N, 54) = 2$

**Answer (3000)**

Sol.  $N$  should be divisible by 2 but not by 3.

$N =$  (Number of numbers divisible by 2) - (Number of numbers divisible by 6)

$$= \frac{9000}{2} - \frac{9000}{6} = 3000$$

22. If  $f(1) + 2f(2) + 3f(3) + \dots + nf(n) = n(n+1)f(n)$

and  $f(1) = 1$ , then  $\frac{1}{f(2022)} + \frac{1}{f(2028)} =$

**Answer (4050)**

Sol.  $f(1) + 2f(2) + \dots + nf(n) = n(n+1)f(n)$  ... (i)

$$f(1) + 2f(2) + \dots + nf(n) + (n+1)f(n+1)$$

$$= (n+1)(n+2)f(n+1) \dots (ii)$$

Using (i) in (ii),

$$n(n+1)f(n) + (n+1)f(n+1) = (n+1)(n+2)f(n+1)$$

$$\Rightarrow f(n+1) = \frac{n}{n+1}f(n)$$

$$f(1) = 1$$

$$\Rightarrow f(2) = \frac{1}{2}, f(3) = \frac{1}{3}, \dots, f(n) = \frac{1}{n}$$

$$\frac{1}{f(2022)} + \frac{1}{f(2028)} = 2022 + 2028 = 4050$$

23. A line  $x + y = 3$  cuts the circle having center (2, 3) and radius 4 at two points  $A$  and  $B$ . Tangents drawn at  $A$  and  $B$  intersect at  $(\alpha, \beta)$ . Find the value of  $4\alpha - 7\beta$ .

**Answer (11)**

Sol. The given line is the polar of  $(\alpha, \beta)$  w.r.t. given circle

$$\text{Circle : } x^2 + y^2 - 4x - 6y - 3 = 0$$

Chord of contact :

$$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$$

$$(\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$$

But equation of chord of contact is

$$x + y - 3 = 0$$

Comparing the coefficients,

$$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{-(2\alpha + 3\beta + 3)}{-3}$$

$$\Rightarrow \alpha = -6, \beta = -5$$

$$\Rightarrow 4\alpha - 7\beta = 11$$

24. Consider a sequence  $a_1, a_2, \dots, a_n$  given by

$a_n = a_{n-1} + 2^{n-1}, a_1 = 1$  and another sequence given by  $b_n = b_{n-1} + a_{n-1}, b_1 = 1$ . Also

$P = \sum_{n=0}^{10} \frac{b_n}{2^{n-1}}$  and  $Q = \sum_{n=0}^{10} \frac{n}{2^{n-1}}$ , then  $2^7(P - 2Q)$  is

**Answer (07.50)**

<b>Sol.</b> $a_2 - a_1 = 2^1$	$b_2 - b_1 = a_1$
$a_3 - a_2 = 2^2$	$b_3 - b_2 = a_2$
$\vdots$	$\vdots$
$a_n - a_{n-1} = 2^{n-1}$	$b_n - b_{n-1} = a_{n-1}$
$a_n = 2^n - 1$	$b_n = 2^n - n$

$$P - 2Q = \sum_{n=1}^{10} \frac{2^n - n}{2^n} - \frac{2n}{2^{n-1}}$$

$$= \sum_{n=1}^{10} \left(1 - \frac{5n}{2^n}\right) = 10 - 5 \sum_{n=1}^{10} \frac{n}{2^n}$$

Let  $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n}{2^n}$

$$\frac{S}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \dots + \frac{n-1}{2^n} + \frac{n}{2^{n+1}}$$

$$\frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n}\right) - \frac{n}{2^{n+1}}$$

$$\Rightarrow \frac{S}{2} = \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{-\frac{1}{2}} - \frac{n}{2^{n+1}}$$

$$\Rightarrow S = 2 \left(1 - \left(\frac{1}{2}\right)^{n+1} - \frac{n}{2^{n+1}}\right)$$

$$= 2 \left(1 - \left(\frac{1}{2}\right)^{10} - \frac{10}{2^{11}}\right) = 2 \left(1 - \frac{12}{2^{11}}\right)$$

$$P - 2Q = 10 - 5 \times 2 \left(1 - \frac{12}{2^{11}}\right)$$

$$= 10 - 10 + \frac{120}{2^{11}} = \frac{60}{2^{10}}$$

$$\Rightarrow 2^7(P - 2Q) = \frac{60}{8} = 07.50$$

- 25.
- 26.
- 27.
- 28.
- 29.
- 30.

