29/01/2023
Evening

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A force $F = -40x$ acts on a mass of 1 kg. x is the position of the mass. If maximum speed of the mass is 4 m/s, find the amplitude. All parameters are in SI units.

(1) $\frac{1}{\sqrt{10}}$ m (2) $\frac{2}{\sqrt{10}}$ m

(3) $\frac{3}{\sqrt{10}}$ m (4) $\frac{4}{\sqrt{10}}$ m

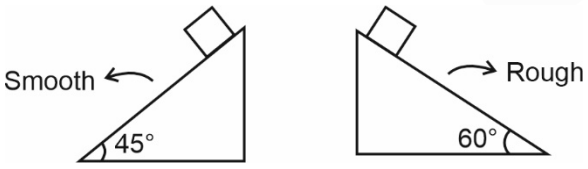
Answer (2)

Sol. $V_{max} = A\omega = A\sqrt{\frac{k}{m}}$

$\Rightarrow \quad 4 = A\sqrt{\frac{40}{1}}$

$\Rightarrow \quad A = \frac{2}{\sqrt{10}}$ m

2. Consider 2 inclined plane of same height. 1st has a smooth surface and angle of inclination is 45°, other has a rough surface and angle of inclination is 60°. If ratio of time taken to slide on then its 'n'. Find coefficient of friction of rough inclined plane.



(1) $\mu = 3n^2$

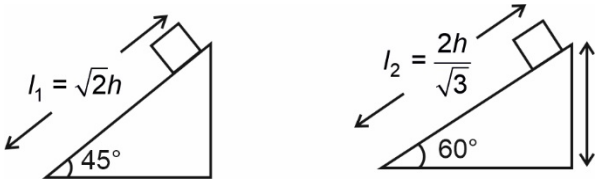
(2) $\mu = \frac{3-2n^2}{\sqrt{3}}$

(3) $\mu = \frac{3-\sqrt{3}n^2}{2}$

(4) $\mu = \frac{2n^2}{\sqrt{3}}$

Answer (2)

Sol.



$a = g\sin\theta = \left(\frac{g}{\sqrt{2}}\right)$

$a = g\sin\theta - \mu g\cos\theta$

$= \left(g\frac{\sqrt{3}}{2} - \frac{\mu g}{2}\right)$

$= g\left(\frac{\sqrt{3}}{2} - \frac{\mu}{2}\right)$

$t_1 = \sqrt{\frac{2l_1}{a}}$

$= \sqrt{\frac{2\sqrt{2}h}{\left(\frac{g}{\sqrt{2}}\right)}}$

$= \sqrt{\frac{4h}{g}}$

$t_2 = \sqrt{\frac{2l_2}{a}}$

$= \sqrt{\frac{2\times 2h}{\sqrt{3}g\left(\frac{\sqrt{3}}{2} - \frac{\mu}{2}\right)}}$

$= \sqrt{\frac{8h}{g\left(3-\sqrt{3}\mu\right)}}$

$\frac{t_1}{t_2} = \sqrt{\frac{3-\sqrt{3}\mu}{2}} = n$

$\Rightarrow 3 - \sqrt{3}\mu = 2n^2$

$\Rightarrow \sqrt{3}\mu = 3 - 2n^2$

$\Rightarrow \mu = \left(\frac{3-2n^2}{\sqrt{3}}\right)$

3. A particle undergoing uniform circular motion about origin. At certain instant $x = 2$ m and $v = -4\hat{j}$ m/s, find velocity and acceleration of particle when at $x = -2$ m.

(1) $\vec{v} = -4\hat{j}$ m/s
$\vec{a} = 8\hat{i}$ m/s^2

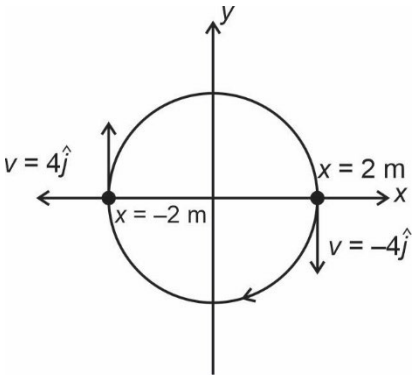
(2) $\vec{v} = 4\hat{j}$ m/s
$\vec{a} = 8\hat{i}$ m/s^2

(3) $\vec{v} = -4\hat{j}$ m/s
$\vec{a} = -8\hat{i}$ m/s^2

(4) $\vec{v} = 4\hat{j}$ m/s
$\vec{a} = -8\hat{i}$ m/s^2

Answer (2)

Sol. For uniform circular motion,



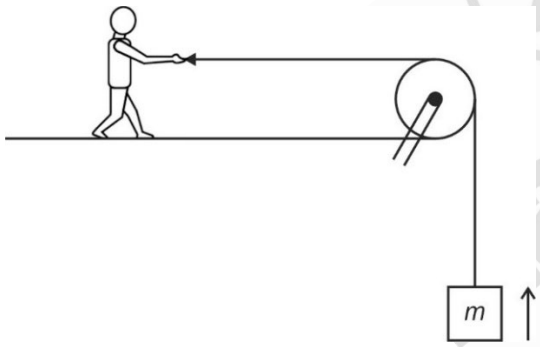
At $x = -2$ m, $v = 4\hat{j}$

$a = \left(\frac{v^2}{R}\right)$ towards the centre

$a = \left(\frac{4^2}{2}\right) = 8\ \text{m/s}^2$

$\vec{a} = 8\ \text{m/s}^2\left(\hat{i}\right)$

4. A man pulls a block as shown:



Consider the following statements:

(a) Work done by gravity on block is +ve

(b) Work done by gravity on block is -ve

(c) If man pulls block with constant speed, then tension in string equals weight of block.

(d) None of the above

Which of the statement(s) is/are correct?

(1) (b) & (c) only (2) (d) only

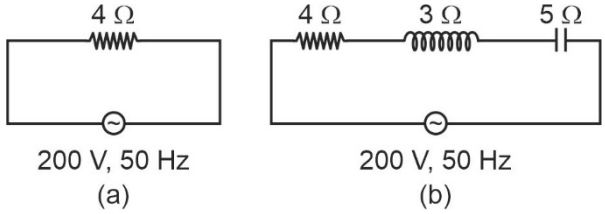
(3) (a) & (c) only (4) (a) only

Answer (1)

Sol. Weight acts down and displacement is up $\Rightarrow$ statement (b) is correct.

$T - mg = ma$

$\Rightarrow$ If $a = 0$, $T = mg$

5. 

RMS current in circuit (a) is I_a while RMS current in circuit (b) is I_b then

(1) $I_a > I_b$ (2) $I_a < I_b$

(3) $I_a = I_b$ (4) None of the above

Answer (1)

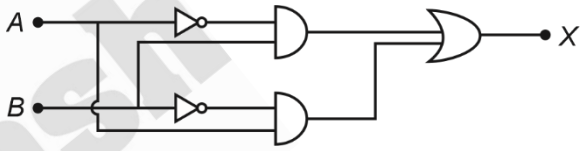
Sol. $Z_a = 4\ \Omega$ & $Z_a = \sqrt{4^2 + (5-3)^2}\,\Omega = \sqrt{20}\,\Omega$

$\Rightarrow I_a = \frac{220}{4}$ & $I_b = \frac{220}{\sqrt{20}}$

$= 55\ A$ $= \frac{110}{\sqrt{5}}$

So $I_a > I_b$

6. Find truth table



(1)

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

(3)

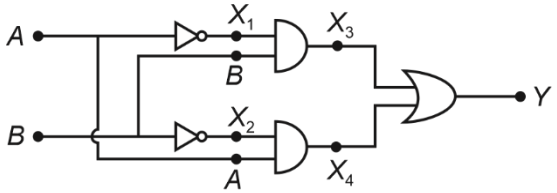
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Answer (4)

Sol.



$X_1 = \bar{A}$

$X_3 = B \cdot \bar{A}$

$X_2 = \bar{B}$

$X_4 = (A\bar{B})$

$Y = X_3 + X_4$

$= A\bar{B} + B\bar{A}$

Y = output = XOR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

7. In a communication system, maximum voltage is 14 mV and minimum voltage is 6 mV. Find out the modulation index.

(1) 0.2 (2) 0.6

(3) 0.4 (4) 0.3

Answer (3)

Sol. Index $= \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{14-6}{14+6} = 0.4$

8. The gravitational potential due to a solid uniform sphere of mass M and radius R at a point at radial distance r ($r > R$) from its centre is equal to

(1) $-\frac{GM}{r}$ (2) $-\frac{GM}{2r}$

(3) $-\frac{GMR}{r^2}$ (4) $-\frac{GM(R+r)}{r^2}$

Answer (1)

Sol. $E_{(r)} = \frac{GM}{r^2} \quad (r > R)$

$dV = -\vec{E} \cdot d\vec{r}$

$\int_V^0 dV = -\int_r^\infty \frac{GM}{r^2} dr$

$V = -\frac{GM}{r}$

9. Resolving power of compound microscope will increase with

(1) Decrease in wavelength of light and increase in numerical aperture

(2) Increase in wavelength of light and decrease in numerical aperture

(3) Increase in both wavelength and numerical aperture

(4) Decrease in both wavelength and numerical aperture

Answer (1)

Sol. Resolving power of microscope $\propto \left(\frac{2n\sin\theta}{\lambda}\right)$

n = Refractive index of the medium separating object and aperture.

$n\sin\theta$ = Numerical aperture

λ = wavelength of light used.

10. It is given that $x^2 + y^2 = a^2$, where a : radius. Also, it is given that $(x - \alpha t)^2 + \left(y - \frac{t}{\beta}\right)^2 = a^2$, where t = time. Then dimensions of α and β are

(1) $[M^0LT^{-1}]$ and $[M^0L^{-1}T]$

(2) $[M^0LT]$ and $[M^0L^{-1}T^{-1}]$

(3) $[M^0LT]$ and $[M^0LT^{-1}]$

(4) $[M^0L^{-1}T]$ and $[M^0LT]$

Answer (1)

Sol. $x \equiv \alpha t = \frac{t}{\beta}$

$\Rightarrow L' \equiv \alpha T' \equiv \frac{T'}{\beta}$

$\Rightarrow \alpha \equiv LT^{-1}$ and $\beta = L^{-1}T$

11. **Assertion (A):** EM waves are not deflected by electric field and magnetic field.

Reason (R): EM waves don't carry any charge so they are not deflected by electric field and magnetic field.

(1) Both (A) and (R) are true and (R) is correct explanation of (A)

(2) Both (A) and (R) are true, but (R) is not correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Answer (1)

Sol. EM wave does not have charge therefore they are not deflected by electric or magnetic field.

12. de-Broglie wavelength of a body of mass m and kinetic energy E is given by

(1) $\lambda = \frac{h}{mE}$ (2) $\lambda = \frac{\sqrt{2mE}}{h}$

(3) $\lambda = \frac{h}{\sqrt{2Em}}$ (4) $\lambda = \sqrt{\frac{h}{2mE}}$

Answer (3)

Sol. $E = \left(\frac{P^2}{2m}\right)$

where P is linear momentum, E = kinetic energy, m = mass of particle.

$P = \sqrt{2Em}$

$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2Em}}$

13. In a region with electric field $30\hat{i}$ V/m a charge particle of charge $q = 2 \times 10^{-4}$ C is displaced slowly from (1, 2) to origin. The work done by the external agent is equal to

(1) 1 mJ (2) 6 mJ

(3) 2 mJ (4) 3 mJ

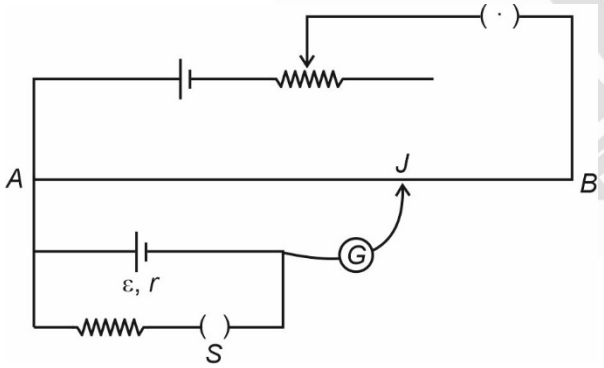
Answer (2)

Sol. $F = qE = 2 \times 10^{-4} \times 30$ N

Work done $= 6 \times 10^{-3} \times (1)$ J $= 6$ mJ

14. Consider the following potentiometer circuit :

When switch S is open, length AJ is 300 cm. When switch S is closed, length AJ is 200 cm. If R 5 Ω, find internal resistance r of the cell.



(1) 4 Ω (2) 2 Ω

(3) 5 Ω (4) 2.5 Ω

Answer (4)

Sol. $C \times 300 = \varepsilon$ ...(1)

$C \times 200 = \frac{\varepsilon}{R+r}R$ ...(2)

$\Rightarrow \frac{300}{200} = \frac{R+r}{R}$

$\Rightarrow r = \frac{R}{2} = 2.5\ \Omega$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. At 300 K, RMS speed of an ideal gas molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times the average speed of gas molecules, then value of α is equal to $\left(\text{take } \pi = \frac{22}{7}\right)$

Answer (28.00)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

$v_{av} = \sqrt{\frac{8RT}{\pi M_0}}$

$\frac{v_{rms}}{v_{av}} = \sqrt{\frac{3\pi}{8}}$

$= \sqrt{\frac{3 \times 22}{8 \times 7}}$

$= \sqrt{\frac{33}{28}} = \sqrt{\frac{28+5}{28}}$

$\Rightarrow \alpha = 28$

22. An α-particle and a proton are accelerated through same potential difference. The ratio of de-Broglie wavelength of alpha particle to proton is equal to $\frac{1}{\sqrt{x}}$. Value of x is (take $m_\alpha = 4m_{proton}$)

Answer (08.00)

Sol. $\lambda = \frac{h}{p}$

$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \sqrt{\frac{1}{4} \times \frac{1}{2}} = \frac{1}{\sqrt{8}}$

$\Rightarrow x = 8$

23. Time period of rotation of a planet is 24 hours. If the radius decreases to $\frac{1}{4}$th of original value, then the new time period is x hours. Find $2x$.

Answer (03.00)

Sol. $I\omega$ = constant

$\Rightarrow \quad I_1\omega_1 = \frac{I_1}{16}\omega_2$

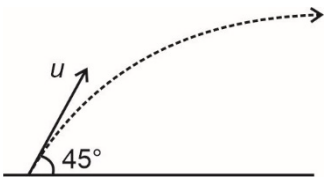
$\Rightarrow \quad \omega_2 = 16\omega_1$

$\Rightarrow \quad T_2 = \frac{T_1}{16} = 1.5$ hours

24. A projectile is fire with velocity 54 km/hr making angle 45° with horizontal. Angular momentum of this particle of mass 1 kg about the point of projection one second into the motion will be $\frac{5N}{\sqrt{2}}$ in SI unit (g = 10 m/s^2). Find the value of N.

Answer (15.00)

Sol. u = 54 km/hr = 15 m/sec.



torque at time t is $\tau = mgu\cos\theta t$

$\frac{dl}{dt} = \tau$

$\int_1^L dL = \int_0^1 mgu\cos\theta t dt$

$L = \frac{mgu\cos\theta}{2} = \frac{10 \times 15}{2\sqrt{2}} = \frac{75}{2}$ kg m^2/sec

So N = 15

25. A block of mass 20 kg is moved with a constant force 'F' for 20 seconds starting from rest and then F is removed. It is then observed that block moves 50 m in next 10 seconds. Find F (in N).

Answer (05.00)

Sol. Impulse, $Ft = mv$

$\Rightarrow \quad v = \frac{50}{10} = 5$ m/s

$F \times 20 = 20 \times 5 \Rightarrow F = 5$ N

26. Atomic mass number of a nuclei A is 16 and half life is 1 day. The values for a nuclei B are 32 and $\frac{1}{2}$ days. 320 grams each of A and B are taken initially. Find the ratio of their number of atoms after 2 days.

Answer (08.00)

Sol. $N_A = N_{0A}e^{-\lambda_A t}$

$N_B = N_{0B}e^{-\lambda_B t}$

$\Rightarrow \quad \frac{N_A}{N_B} = \frac{N_{0A}}{N_{0B}}\frac{e^{-\lambda_A t}}{e^{-\lambda_B t}}$

$= \frac{\frac{320}{16}}{\frac{320}{32}} \times \frac{\frac{1}{4}}{\frac{1}{16}}$

$= 8$

27.

28.

29.

30.

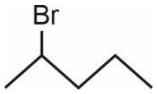
CHEMISTRY

SECTION – A

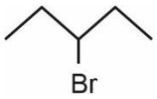
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

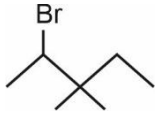
Choose the correct answer :

1. In which molecule, dehydrohalogenation forms maximum number of isomers (excluding rearrangement)

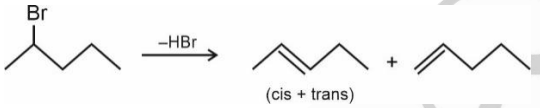
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. (1) 

Total 3 isomers are possible

(2) → Only 1 product

(3) → 2 products

(4) → Only 1 product

2. Which of the following complex has zero spin only magnetic moment?

(1) $\left[Fe(F)_6\right]^{3-}$

(2) $\left[CoF_6\right]^{3-}$

(3) $\left[Co(C_2O_4)_3\right]^{3-}$

(4) $\left[Fe(H_2O)_6\right]^{3+}$

Answer (3)

Sol. $\left[Co(C_2O_4)_3\right]^{3-}$ has d^2sp^3 hybridisation and $3d^6$ electronic configuration and it has zero unpaired electrons.

3. Which of the following diseases can be cured by equanil drug?

(1) Pain (2) Stomach ulcer

(3) Depression (4) Hyperacidity

Answer (3)

Sol. Based on fact

4. If Bohr's Radius of H-atom in Ground state is 0.6 A°. Find out Bohr's Radius of 3rd orbit of He^+ Ion.

(1) 2.7 A° (2) 0.9 A°

(3) 5.4 A° (4) 1.8 A°

Answer (1)

Sol. $r \propto \frac{n^2}{z}$

$$\boxed{r = \frac{.6 \times n^2}{z}}$$

$$r = \frac{.6 \times (3)^2}{(2)}$$

$= .3 \times 9$

$= 2.7$ A°

5. Compare the bond order of the following molecules

O_2^{-2}, NO, CO

(1) $O_2^{-2} > NO > CO$ (2) $O_2^{-2} > CO > NO$

(3) $CO > NO > O_2^{-2}$ (4) $NO > CO > O_2^{-2}$

Answer (3)

Sol. The correct bond order :

$O_2^{-2} \rightarrow 1$

$CO \rightarrow 3$

$NO \rightarrow 2.5$

$\therefore$ Correct order is $CO > NO > O_2^{-2}$

6. Which one of the following ores contains sulphide ions?

(1) Malachite (2) Calamine

(3) Sphalerite (4) Siderite

Answer (3)

Sol. The chemical formulae of the given ores are

Malachite : $CuCO_3.Cu(OH)_2$

Calamine : $ZnCO_3$

Sphalerite : ZnS

Siderite : $FeCO_3$

$\therefore$ Sphalerite contains sulphide ions.

7. Statement-I : Ionisation enthalpy difference from B to Al is more than that of Al to Ga.

Statement-II : Ga has completely filled d-orbital.

Then, the correct option is?

(1) Statement-I and Statement-II both are correct.

(2) Statement-I is incorrect and Statement-II is correct.

(3) Statement-I is correct and Statement-II is incorrect.

(4) Statement-I and Statement-II both are incorrect.

Answer (1)

Sol. Ga has similar ionisation enthalpy as Al because of inert pair effect (or completely filled d-orbital in Ga).

8. Which of the following relation is correct?

(1) $\Delta G = \Delta H - T\Delta S$ (at constant T& P)

(2) $\Delta U = \Delta H + nR\Delta T$ (for n moles of ideal gas)

(3) $P\Delta V = (\Delta n)RT$

(4) None of these

Answer (1)

Sol. $\Delta G = \Delta H - T\Delta S \rightarrow$ correct relation at constant T & P

$\Delta H = \Delta U + nR\Delta T$ (for n moles of an ideal gas)

$P\Delta V = (\Delta n)RT$ [is only true for a chemical reaction at constant T & P] (not always true)

So, correct answer is (1)

9. Match the correct column.

(A) Thermosetting	(p) Neoprene
(B) Thermoplastic	(q) Polyester
(C) Elastomer	(r) Polystyrene
(D) Fibre	(s) Urea formaldehyde resin

(1) $A \rightarrow p; B \rightarrow r; C \rightarrow q; D \rightarrow s$

(2) $A \rightarrow s; B \rightarrow r; C \rightarrow p; D \rightarrow q$

(3) $A \rightarrow s; B \rightarrow r; C \rightarrow q; D \rightarrow p$

(4) $A \rightarrow p; B \rightarrow r; C \rightarrow s; D \rightarrow q$

Answer (2)

Sol.
- Urea- formaldehyde resin is Thermosetting
- Polystyrene is Thermoplastic

10. At 300 K the ratio of V_{rms} and V_{avg} of oxygen molecule is $\sqrt{\frac{\alpha\pi}{\alpha+5}}$, the value of α will be

(1) 1 (2) 2

(3) 3 (4) 4

Answer (3)

Sol. $\frac{V_{rms}}{V_{avg}} = \frac{\sqrt{3\pi}}{\sqrt{8}} = \sqrt{\frac{\alpha\pi}{\alpha+5}}$

11. Thermal decomposition products of $LiNO_3$ are

$$LiNO_3 \xrightarrow{\Delta} \text{Products}$$

(1) $LiNO_2$ and O_2

(2) $LiNO_2$, NO_2 and O_2

(3) Li_2O, NO_2 and O_2

(4) Li, NO and O_2

Answer (3)

Sol. Thermal decomposition of $LiNO_3$ gives the following products

$$4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$$

12. BOD value of drinking water ranges between

(1) 3-5

(2) 10-13

(3) 14-17

(4) 20-22

Answer (1)

Sol. BOD value of drinking water ranges between 3 and 5

13. Match List-I with List-II.

(A) Electro-osmosis	(P) Solvent moves from low concentration to high concentration of solution
(B) Electrophoresis	(Q) Solvent moves from high concentration to low concentration of solution
(C) RO(Reverse osmosis)	(R) Dispersion medium (DM) moves towards oppositely charged electrode across semi-permeable membrane
(D) Osmosis	(S) Colloidal particles move in the presence of electric field. (DP and DM)

(1) A(R), B(S), C(Q), D(P)

(2) A(Q), B(P), C(R), D(S)

(3) A(P), B(Q), C(R), D(S)

(4) A(P), B(R), C(Q), D(S)

Answer (1)

Sol. All options are definition based.

(A) Electro-osmosis $\rightarrow$ movement of DM across SPM

(B) Electrophoresis $\rightarrow$ movement of DP and DM towards respective electrodes

(C) RO $\rightarrow$ movement of solvent from high concentration to low concentration

(D) Osmosis $\rightarrow$ movement of solvent from low concentration to high concentration

14. The ratio of de-Broglie wavelength of proton to that of α-particle if they are accelerated through same potential

(1) $2\sqrt{2}:1$

(2) $2:1$

(3) $1:2\sqrt{2}$

(4) $\sqrt{2}:1$

Answer (1)

Sol. $\frac{\lambda\rho}{\lambda\alpha}=\sqrt{\frac{m\alpha\cdot kE_\alpha}{m\rho\cdot kE_\rho}}$

$=\sqrt{\frac{4m\rho\cdot 2V}{m\rho\cdot V}}$

$=\sqrt{8}:1$

$=2\sqrt{2}:1$

15. Which of the following is produced when propanamide is treated with Br_2 in presence of KOH?

(1) Ethyl nitrile

(2) Propanamine

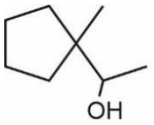
(3) Ethylamine

(4) Propanenitrile

Answer (3)

Sol. $CH_3CH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow{Br_2/KOH} CH_3CH_2NH_2$

16. Consider the following reaction:



Major product 'P'

Find the number of α-H in the major product 'P'?

(1) 7

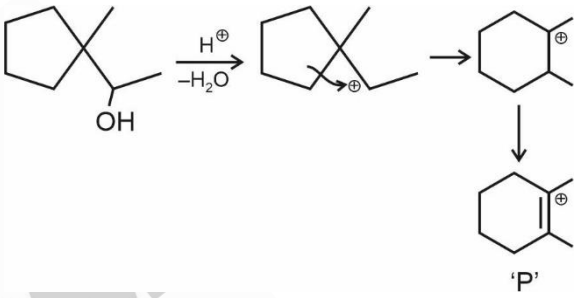
(2) 8

(3) 9

(4) 10

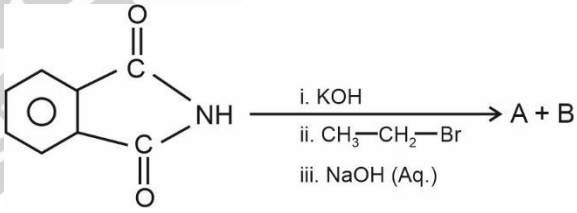
Answer (4)

Sol.

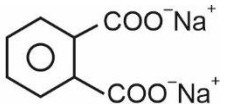


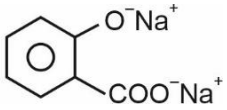
Number of α-H in 'P' = 10

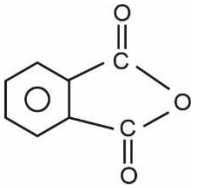
17.

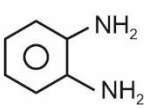


A and B respectively

(1) $CH_3-CH_2-NH_2$ and 

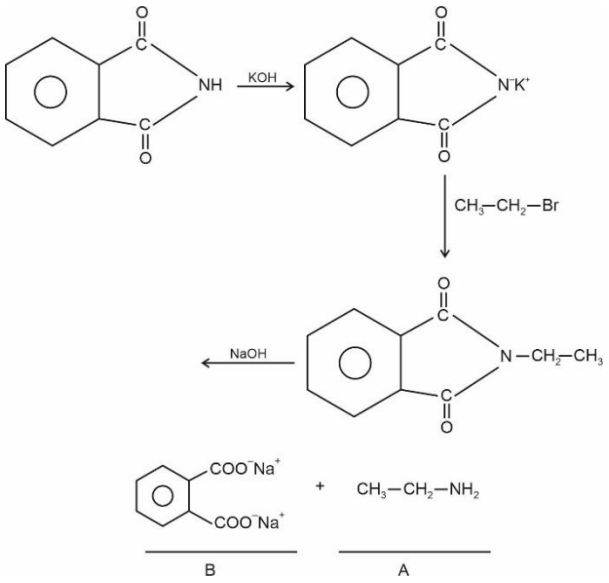
(2) $CH_3-CH_2-NH_2$ and 

(3) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2$ and 

(4) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-OH$ and 

Answer (1)

Sol.



18. The colour of CrO_5 in ether is

(1) Yellow

(2) Green

(3) Blue

(4) Orange

Answer (3)

Sol. CrO_5 is blue in colour

19. The number of voids in 0.02 moles of a solid which forms HCP lattice is _____.

[Given : $N_A = 6 \times 10^{23}$]

(1) 3.6×10^{22}

(2) 3.6×10^{24}

(3) 7.2×10^{20}

(4) 5.4×10^{26}

Answer (1)

Sol. Voids $= \frac{18}{6} \times 6 \times 10^{23} \times 0.02$

$= 3.6 \times 10^{22}$

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. How many oxides are acidic?

$NO, NO_2, N_2O_3, Cl_2O_7, CO, SO_2, SO_3, N_2O$

Answer (5)

Sol. Acidic oxides $\rightarrow NO_2, N_2O_3, Cl_2O_7, SO_2, SO_3$

22. A 1 : 1 (by mole) mixture of A and B is present in a container. Molar mass of A = 16 g and molar mass of B is 32 and the half life of A is 1 day and half life of B is $\frac{1}{2}$ day. Then find the average molar mass of the mixture of A and B remained in the container after 2 days is _____.? [Round off to nearest integer]

Answer (19)

Sol. For A $1 \xrightarrow{2 \text{ days}} \frac{1}{4}$ moles remained

For B $1 \xrightarrow{2 \text{ days}} \frac{1}{16}$ moles remained

$\therefore \quad M_{avg} = \dfrac{\frac{1}{4} \times 16 + \frac{1}{16} \times 32}{\frac{1}{4} + \frac{1}{16}}$

$= 19.2$ g

23.

24.

25.

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The 3 digit numbers which are divisible either 3 or 4 but not divisible by 48

(1) 414 (2) 420

(3) 429 (4) 432

Answer (4)

Sol. No's divisible by 3 = 300

No's divisible by 4 = 225

No's divisible by 12 = 75

No's divisible by 48 = 18

Total no's = 300 + 225 – 75 – 18

= 432

2. The letters of the word GHOTU is arranged alphabetically as in a dictionary. The rank of the word TOUGH is

(1) 84 (2) 79

(3) 74 (4) 89

Answer (4)

Sol. GHOTU

G _ _ _ _ 24

H _ _ _ _ 24

O _ _ _ _ 24

T G _ _ _ 6

T H _ _ _ 6

T O G _ _ 2

T O H _ _ 2

T O U G H 1

= 89

3. $\int_{1/2}^{2} \frac{\tan^{-1}x}{x}dx$ equals

(1) $\frac{\pi}{2}\ln 2$ (2) $\frac{\pi}{4}\ln 2$

(3) $\pi \ln 2$ (4) $\ln 2$

Answer (1)

Sol. $I = \int_{1/2}^{2} \frac{\tan^{-1}x}{x}dx$

Put $x = \frac{1}{t}$

$$I = \int_{1/2}^{2} \frac{\tan^{-1}}{t}\left(\frac{1}{t}\right)dt$$

$$\therefore \quad 2I = \int_{1/2}^{2} \frac{\pi}{2}\,\frac{dt}{t}$$

$$\Rightarrow \quad I = \frac{\pi}{2}\ln 2$$

4. Shortest distance between lines:

$\frac{x-1}{2} = \frac{2y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z+2}{4}$ is

(1) $\frac{13}{\sqrt{165}}$ (2) $\frac{15}{\sqrt{165}}$

(3) $\frac{18}{\sqrt{165}}$ (4) $\frac{19}{\sqrt{165}}$

Answer (1)

Sol. Lines are

$$\frac{x-1}{2} = \frac{y-1}{\frac{3}{2}} = \frac{z-3}{1} \;\&\; \frac{x-2}{3} = \frac{y-1}{2} = \frac{z+2}{4}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & \frac{3}{2} & 1 \\ 3 & 2 & 4 \end{vmatrix} = \hat{i}(4) - \hat{j}(5) + \hat{k}\left(-\frac{1}{2}\right)$$

$$\left(\vec{a_2}-\vec{a_1}\right)\cdot\left(\vec{b_1}\times\vec{b_2}\right) = \left(\hat{i}-5\hat{k}\right)\left(4\hat{i}-5\hat{j}-\frac{\hat{k}}{2}\right)$$

$$= 4 + \frac{5}{2} = \frac{13}{2}$$

$$\therefore \quad \text{Shortest distance} = \frac{\frac{13}{2}}{\sqrt{16+25+\frac{1}{4}}} = \frac{3}{\sqrt{165}}$$

5. $R = \{(a, b) : 2a + 3b$ is divisible by 5 and $a, b \in N\}$ is

(1) Transitive but not symmetric

(2) Equivalence relation

(3) Symmetric but not transitive

(4) Not equivalence

Answer (2)

Sol. Let $(a, b) \in R$

$f(a, b) = 2a + 3b$

For Reflexive

$f(a, a) = 2a + 3a = 5a$ *i.e*, divisible by 5

$\Rightarrow (a, a) \in R$

For symmetric

$f(b, a) = 2b + 3a = \underbrace{5a + 5b}_{\text{divisible by 5}} - \underbrace{(2a + 3b)}_{\text{divisible by 5}}$

$f(b, a)$ is divisible by 5

$\Rightarrow (b, a) \in R$

For transitive

$f(a, b) = 2a + 3b$ is divisible by 5

$f(b, c) = 2b + 3c$ is divisible by 5

$\Rightarrow 2a + 5b + 3c$ is divisible by 5

So, $2a + 3c$ is divisible by 5

$\Rightarrow (a, c) \in R$

6. $(\sim A) \vee B$ is equivalent to

(1) $A \rightarrow B$

(2) $A \leftrightarrow B$

(3) $\sim A \wedge B$

(4) $B \rightarrow A$

Answer (1)

Sol. Making truth table

A	B	$\sim B$	$(\sim A) \vee B$	$A \rightarrow B$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The truth table clearly shows.

$(\sim A) \vee B \equiv A \rightarrow B$

7. The value of $\int_{\frac{1}{2}}^{2}\left(\frac{t^4+1}{t^6+1}\right)dt =$

(1) $\tan^{-1}2 + \tan^{-1}8 + \frac{2\pi}{3}$

(2) $2\tan^{-1}2 + \frac{2}{3}\tan^{-1}8 - \frac{2\pi}{3}$

(3) $2\tan^{-1}2 + \frac{2}{3}\tan^{-1}8 + \frac{2\pi}{3}$

(4) $2\tan^{-1}2 - \frac{2}{3}\tan^{-1}(8) + \frac{2\pi}{3}$

Answer (2)

Sol. $= \int_{\frac{1}{2}}^{2}\frac{t^4+1}{t^6+1}dt$

$= \int_{\frac{1}{2}}^{2}\frac{\left(t^4+1\right)\left(t^2+1\right)}{\left(t^6+1\right)\left(t^2+1\right)}dt = \int_{\frac{1}{2}}^{2}\frac{t^6+1+t^2\left(t^2+1\right)}{\left(t^6+1\right)\left(t^2+1\right)}dt$

$= \int_{\frac{1}{2}}^{2}\frac{dt}{t^2+1} + \frac{1}{3}\int_{\frac{1}{2}}^{2}\frac{3t^2dt}{t^6+1}$

$= \tan^{-1}t\Big|_{\frac{1}{2}}^{2} + \frac{1}{3}\tan^{-1}\left(t^3\right)\Big|_{\frac{1}{2}}^{2}$

$= \left(\tan^{-1}2 - \tan^{-1}\left(\frac{1}{2}\right)\right) + \frac{1}{3}\left(\tan^{-1}(8) - \tan^{-1}\left(\frac{1}{8}\right)\right)$

$= \left(\tan^{-1}2 - \cot^{-1}2\right) + \frac{1}{3}\left(\tan^{-1}8 - \cot^{-1}8\right)$

$= \tan^{-1}2 - \left(\frac{\pi}{2} - \tan^{-1}2\right) + \frac{1}{3}\left(\tan^{-1}8 - \left(\frac{\pi}{2} - \tan^{-1}8\right)\right)$

$= 2\left(\tan^{-1}2\right) + \frac{2}{3}\tan^{-1}8 - \frac{\pi}{2} - \frac{\pi}{6}$

$= 2\tan^{-1}2 + \frac{2}{3}\tan^{-1}8 - \frac{2\pi}{3}$

8. Area of region

$|\cos x - \sin x| \le y \le \sin x$ for $x \in (0, \pi/2)$ is

(1) $\left(-1 + 2\sqrt{2}\right)$ sq. unit

(2) $\left(1 - \frac{1}{\sqrt{2}}\right)$ sq. unit

(3) $\left(\sqrt{5} + 1 - 2\sqrt{2}\right)$ sq. unit

(4) $\left(\sqrt{5} - \sqrt{2}\right)$ sq. unit

Answer (3)

Sol. $A = \int_{\theta}^{\pi/2} (\sin x - |\cos x - \sin x|)dx$

Where $\theta = \tan^{-}\frac{1}{2}$

$= \int_{\theta}^{\pi/4} (\sin x - \cos x + \sin x)dx +$

$\int_{\pi/4}^{\pi/2} (\sin x + \cos x - \sin x)dx$

$= -2\cos x - \sin x|_{\theta}^{\pi/4} + \left(1 - \frac{1}{\sqrt{2}}\right)$

$= -\left(\sqrt{2} + \frac{1}{\sqrt{2}} - 2\cos\theta - \sin\theta\right) + 1 - \frac{1}{\sqrt{2}}$

$= -\sqrt{2} - \frac{1}{\sqrt{2}} + (2\cos\theta + \sin\theta) + 1 - \frac{1}{\sqrt{2}}$

$= 1 - 2\sqrt{2} + 2\cdot\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}}$

$= \sqrt{5} + 1 - 2\sqrt{2}$

9. For solution of differential equation $x\ln x\frac{dy}{dx} + y = x^2\ln x, y(2) = 2$ then $y(e)$ is equal to

(1) $1 + \frac{e^2}{4}$ (2) $1 - \frac{e^2}{4}$

(3) $\frac{e^2}{2}$ (4) $1 + \frac{e^2}{2}$

Answer (1)

Sol. $x\ln x\frac{dy}{dx} + y = x^2\ln x$

$\frac{dy}{dx} + \frac{y}{x\ln x} = x$

I.F = $e^{\int \frac{1}{x\ln x}dx} = e^{\ln|\ln x|} = |\ln x|$

Solution of equation is

$y(IF) = \int x\cdot|\ln x|\,dx$

$y|\ln x| = |\ln x|\frac{x^2}{2} - \frac{x^2}{4} + c$

$\boxed{x = 2}$, $2|\ln 2| = |\ln 2|\cdot 2 - 1 + c$

$\Rightarrow \boxed{c = 1}$

For $x = e$

$y = \frac{e^2}{2} - \frac{e^2}{4} + 1$

$\Rightarrow y(e) = 1 + \frac{e^2}{4}$

10. Let $f(x) = x^2 + 2x + 5$ and α, β be roots of $f\left(\frac{1}{t}\right) = 0$, then $\alpha + \beta =$

(1) $\frac{-2}{5}$ (2) -2

(3) $\frac{5}{2}$ (4) $\frac{-5}{2}$

Answer (1)

Sol. $f(x) = x^2 + 2x + 5$

$f\left(\frac{1}{t}\right) = 0 \Rightarrow \frac{1}{t^2} + \frac{2}{t} + 5 = 0$

$\Rightarrow 5t^2 + 2t + 1 = 0 \quad [t \neq 0]$

This equation has roots α and β.

$\therefore \alpha + \beta = \frac{-2}{5}$

11. If the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{1}$ and $\frac{x-11}{4} = \frac{y-9}{2} = \frac{z-4}{3}$ intersects at point p, then perpendicular distance of p from plane $3x + 2y + 6z = 10$ is

(1) $\frac{2}{7}$ (2) $\frac{3}{7}$

(3) $\frac{4}{7}$ (4) $\frac{5}{7}$

Answer (2)

Sol. Given

$L_1 \equiv \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{1} = \lambda$

and $L_2 \equiv \frac{x-11}{4} = \frac{y-9}{2} = \frac{z-4}{3} = \mu$

Finding intersection, we get $\lambda = 1$, $\mu = -2$

$\therefore p \equiv (3, 5, -2)$

Distance from given plane $= \left|\frac{9 + 10 - 12 - 10}{\sqrt{9 + 4 + 36}}\right| = \frac{3}{7}$

12. If $\cos^2 2x - \sin^4 x - 2\cos^2 x = \lambda$, has a solution $\forall x \in R$, then the range of λ is

(1) $\left[-\frac{1}{2}, 1\right]$ (2) $\left[-\frac{4}{3}, 0\right]$

(3) (0, 2) (4) None of these

Answer (2)

Sol. $\cos^2 2x - \sin^4 x - 2\cos^2 x = \lambda$

$\Rightarrow \left(\cos^2 x - \sin^2 x\right)^2 - \sin^4 x - 2\cos^2 x = \lambda$

$\Rightarrow 3\cos^4 x - 4\cos^2 x = \lambda$

$\Rightarrow 3\left(\left(\cos^2 x - \frac{2}{3}\right)^2 - \frac{4}{9}\right) = \lambda$

$\lambda_{min} = -\frac{4}{3}$

$\lambda_{max} = 0$

$\therefore \quad \lambda \in \left[-\frac{4}{3}, 0\right]$

13. $\vec{a} = 9\hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 7\hat{i} - 3\hat{j} + 2\hat{k}$ are three given vectors. Let there be a $\vec{r}$ such that $(\vec{r} \times \vec{b}) + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$ then $\vec{r} \cdot \vec{c}$ is

(1) $\frac{280}{11}$ (2) 28

(3) $\frac{279}{13}$ (4) $\frac{290}{11}$

Answer (1)

Sol. $\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$(\vec{r} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

$\vec{r} \cdot \vec{a} = 0$

$\vec{c} \cdot \vec{a} + \lambda\vec{b} \cdot \vec{a} = 0$

$67 + \lambda(11) = 0$

$\boxed{\lambda = -\frac{67}{11}}$

$\vec{r} \cdot \vec{c} = (\vec{c} + \lambda\vec{b}) \cdot \vec{c}$

$= |\vec{c}|^2 + \lambda\, \vec{b} \cdot \vec{c}$

$= 62 - \frac{67}{11}(7 - 3 + 2)$

$= 62 - \frac{67}{11}(6)$

$= \frac{682 - 402}{11} = \frac{280}{11}$

14. For observation set x data obtained is

$x_i = \{11, 12, 13, \ldots 41\}$

For another observation set y data obtained is

$y_i = \{61, 62, 63, \ldots 91\}$

Then value of $\bar{x} + \bar{y} + \sigma^2$ where $\bar{x}, \bar{y}$ are means of respective data set while 6^2 is variance of combined data is

(1) 801 (2) 754

(3) 807 (4) 774

Answer (3)

Sol. $\bar{x} = \frac{\frac{31}{2}(11 + 41)}{31} = \frac{1}{2} \times 52 = 26$

$\bar{y} = \frac{\frac{31}{2}(61 + 91)}{31} = \frac{1}{2} \times 152 = 76$

$\sigma^2 = \frac{\Sigma x_i^2 + \Sigma y_i^2}{62} - \left(\frac{\Sigma x_i + \Sigma y_i}{62}\right)^2$

$= \frac{(11^2 + 12^2 + \ldots + 41^2) + (61^2 + 62^2 + \ldots + 91^2)}{62} - (51)^2$

$= \frac{\left(\frac{41 \cdot 42 \cdot 83}{6} - \frac{10 \cdot 11 \cdot 21}{6}\right) + \left(\frac{91 \cdot 92 \cdot 183}{6} - \frac{60 \cdot 61 \cdot 121}{6}\right)}{62} - (51)^2$

$= \frac{(41 \cdot 7 \cdot 83 - 11 \cdot 35) + (91 \cdot 46 \cdot 61 - 10 \cdot 61 \cdot 121)}{62} - (51)^2$

$= \frac{23436 + 181536}{62} - (51)^2$

$= 3306 - 2601 = 705$

$\bar{x} + \bar{y} + \sigma^2$

$= 26 + 76 + 705$

$= 807$

15. If curve represented by $y = \frac{(x - a)}{(x - 3)(x - 2)}$ passes through (1, –3) then equation of normal at (1, –3) to the curve is given by

(1) $2x + 3y = -7$ (2) $3x - 2y = 9$

(3) $3x - 4y = 21$ (4) $x - 4y = 13$

Answer (4)

Sol. If passes through (1, –3)

$$-3 = \frac{1-a}{(-2)(-1)}$$

$$\boxed{a = 7}$$

$$f'(x) = \frac{(x-3)(x-2)-(x-7)(2x-5)}{\left((x-3)(x-2)\right)^2}$$

$$f'(1) = \frac{2-18}{(2)^2} = -4$$

Slope of normal $= \frac{1}{4}$

Equation of normal

$$y + 3 = \frac{1}{4}(x-1)$$

or $4y + 12 = x - 1$

or $x - 4y = 13$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the number of four-digit numbers N such that $GCD(N, 54) = 2$

Answer (3000)

Sol. N should be divisible by 2 but not by 3.

N = (Number of numbers divisible by 2) – (Number of numbers divisible by 6)

$$= \frac{9000}{2} - \frac{9000}{6} = 3000$$

22. If $f(1) + 2f(2) + 3f(3) + ... + nf(n) = n(n+1)f(n)$ and $f(1) = 1$, then $\frac{1}{f(2022)} + \frac{1}{f(2028)} =$

Answer (4050)

Sol. $f(1) + 2f(2) + ... + nf(n) = n(n+1)f(n)$ …(i)

$$f(1) + 2f(2) + ... + nf(n) + (n+1)f(n+1)$$

$$= (n+1)(n+2)f(n+1) \quad \text{…(ii)}$$

Using (i) in (ii),

$$n(n+1)f(n) + (n+1)f(n+1) = (n+1)(n+2)f(n+1)$$

$$\Rightarrow \quad f(n+1) = \frac{n}{n+1}f(n)$$

$$f(1) = 1$$

$$\Rightarrow \quad f(2) = \frac{1}{2},\ f(3) = \frac{1}{3},\ .....,f(n) = \frac{1}{n}$$

$$\frac{1}{f(2022)} + \frac{1}{f(2028)} = 2022 + 2028 = 4050$$

23. A line $x + y = 3$ cuts the circle having center (2, 3) and radius 4 at two points A and B. Tangents drawn at A and B intersect at (α, β). Find the value of $4\alpha - 7\beta$.

Answer (11)

Sol. The given line is the polar of (α, β) w.r.t. given circle

Circle : $x^2 + y^2 - 4x - 6y - 3 = 0$

Chord of contact :

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$(\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$

But equation of chord of contact is

$x + y - 3 = 0$

Comparing the coefficients,

$$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{-(2\alpha + 3\beta + 3)}{-3}$$

$\Rightarrow \quad \alpha = -6, \beta = -5$

$\Rightarrow \quad 4\alpha - 7\beta = 11$

24. Consider a sequence $a_1, a_2 \ldots\ldots, a_n$ given by $a_n = a_{n-1} + 2^{n-1}, a_1 = 1$ and another sequence given by $b_n = b_{n-1} + a_{n-1}$, $b_1 = 1$. Also

$P = \sum_{n=0}^{10} \frac{b_n}{2^{n-1}}$ and $Q = \sum_{n=0}^{10} \frac{n}{2^{n-1}}$, then $2^7(P - 2Q)$ is

Answer (07.50)

Sol.

$$\begin{array}{l|l} a_2 - a_1 = 2^1 & b_2 - b_1 = a_1 \\ a_3 - a_2 = 2^2 & b_3 - b_2 = a_2 \\ \vdots & \vdots \\ a_n - a_{n-1} = 2^{n-1} & b_n - b_{n-1} = a_{n-1} \\ \hline a_n = 2^n - 1 & b_n = 2^n - n \end{array}$$

$$P - 2Q = \sum_{n=1}^{10} \frac{2^n - n}{2^n} - \frac{2n}{2^{n-1}}$$

$$= \sum_{n=1}^{10} \left(1 - \frac{5n}{2^n}\right) = 10 - 5\Sigma \frac{n}{2^n}$$

Let $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{n}{2^n}$

$$\begin{array}{l} \frac{S}{2} = \quad \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{n-1}{2n} + \frac{n}{2^{n+1}} \\ \hline \frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^n}\right) - \frac{n}{2^{n+1}} \end{array}$$

$$\Rightarrow \frac{S}{2} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^n - 1\right)}{-\frac{1}{2}} - \frac{n}{2^{n+1}}$$

$$\Rightarrow S = 2\left(1 - \left(\frac{1}{2}\right)^n - \frac{n}{2^{n+1}}\right)$$

$$= 2\left(1 - \left(\frac{1}{2}\right)^{10} - \frac{10}{2^{11}}\right) = 2\left(1 - \frac{12}{2^{11}}\right)$$

$$P - 2Q = 10 - 5 \times 2\left(1 - \frac{12}{2^{11}}\right)$$

$$= 10 - 10 + \frac{120}{2^{11}} = \frac{60}{2^{10}}$$

$$\Rightarrow 2^7(P - 2Q) = \frac{60}{8}$$

$$= 07.50$$

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