Important Instructions:

1. The test is of 3 hours duration.

2. The Test Booklet consists of 90 questions. The maximum marks are 360.

3. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.

4. Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

5. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction No. 4 above.

6. For writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet use only Black Ball Point Pen provided in the examination hall.

7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.
1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 2.5%  (2) 3.5%  (3) 4.5%  (4) 6%

Answer (3)

Sol. \( \rho = \frac{m}{l^3} \)

\[ \frac{d\rho}{\rho} = \frac{dm}{m} + 3 \frac{dl}{l} \]

\[ = (1.5 + 3 \times 1) \]

\[ = 4.5\% \]

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(1) Velocity  Position

(2) Distance  Time

(3) Position  Time

(4) Velocity  Time

Answer (2)

Sol. Options (1), (3) and (4) correspond to uniformly accelerated motion in a straight line with positive initial velocity and constant negative acceleration, whereas option (2) does not correspond to this motion.

3. Two masses \( m_1 = 5 \text{ kg} \) and \( m_2 = 10 \text{ kg} \), connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight \( m \) that should be put on top of \( m_2 \) to stop the motion is

(1) 18.3 kg  (2) 27.3 kg  (3) 43.3 kg  (4) 10.3 kg

Answer (2)

Sol. To stop the moving block \( m_2 \), acceleration of \( m_2 \) should be opposite to velocity of \( m_2 \)

\[ m_1g < \mu(m + m_2)g \]

\[ 5 < 0.15(10 + m_2) \]

\[ m_2 > 23.33 \text{ kg} \]

\[ \therefore \text{ Minimum mass } = 27.3 \text{ kg (according to given options)} \]

4. A particle is moving in a circular path of radius \( a \) under the action of an attractive potential \( U = \frac{k}{2r^2} \).

Its total energy is

(1) \( \frac{k}{4a^2} \)

(2) \( \frac{k}{2a^2} \)

(3) Zero

(4) \( \frac{3k}{2a^2} \)

Answer (3)
Sol. \( F = -\frac{dU}{dr} \)  
\[
U = -\frac{k}{2r^2}
\]

\[
\frac{mv^2}{r} = \frac{k}{r^3}
\]

[This force provides necessary centripetal force]

\[
\Rightarrow \quad \frac{mv^2}{r} = \frac{k}{r^3}
\]

\[
\Rightarrow \quad K.E = \frac{k}{2r^2}
\]

\[
\Rightarrow \quad P.E = -\frac{k}{2r^2}
\]

Total energy = Zero

5. In a collinear collision, a particle with an initial speed \( v_0 \) strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(1) \( \frac{v_0}{4} \)  
(2) \( \sqrt{2}v_0 \)  
(3) \( \frac{v_0}{2} \)  
(4) \( \frac{v_0}{\sqrt{2}} \)

Answer (2)

Sol. It is a case of superelastic collision

\[
mv_0 = mv_1 + mv_2 \quad \text{(i)}
\]

\[
\Rightarrow \quad v_1 + v_2 = v_0
\]

\[
\frac{1}{2} m(v_1^2 + v_2^2) = \frac{3}{2} \left( \frac{1}{2} mv_0^2 \right)
\]

\[
\Rightarrow \quad (v_1^2 + v_2^2) = \frac{3}{2} v_0^2 \quad \text{(ii)}
\]

\[
\Rightarrow \quad (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2
\]

\[
\Rightarrow \quad v_0^2 = \frac{3v_0^2}{2} + 2v_1v_2
\]

\[
\Rightarrow \quad v_0^2 = \frac{3v_0^2}{2} + 2v_1v_2
\]

\[
\Rightarrow \quad 2v_1v_2 = \frac{-v_0^2}{2} \quad \text{(iii)}
\]

\[
\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v_0^2 + v_0^2
\]

\[
\Rightarrow \quad v_1 - v_2 = \sqrt{2}v_0
\]

6. Seven identical circular planar disks, each of mass \( M \) and radius \( R \) are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point \( P \) is

(1) \( \frac{19}{2} MR^2 \)  
(2) \( \frac{55}{2} MR^2 \)  
(3) \( \frac{73}{2} MR^2 \)  
(4) \( \frac{181}{2} MR^2 \)

Answer (4)

Sol. \( I_0 = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right) \)

\( I_p = I_0 + 7M(3R)^2 \)

\( = \frac{181}{2} MR^2 \)

7. From a uniform circular disc of radius \( R \) and mass \( 9M \), a small disc of radius \( \frac{R}{3} \) is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

(1) \( 4MR^2 \)  
(2) \( \frac{40}{9} MR^2 \)  
(3) \( 10MR^2 \)  
(4) \( \frac{37}{9} MR^2 \)

Answer (1)

Sol. \( m = \frac{(9M)}{9} = M \)

\( I_t = \frac{(9M)R^2}{2} \)
8. A particle is moving with a uniform speed in a circular orbit of radius $R$ in a central force inversely proportional to the $n^{th}$ power of $R$. If the period of rotation of the particle is $T$, then

\[ T \propto R^{\frac{n+1}{2}} \]

Answer (3)

Sol. \[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi R^{\frac{n+1}{2}}}{m}} = R^{\frac{n+1}{2}} \]

9. A solid sphere of radius $r$ made of a soft material of bulk modulus $K$ is surrounded by a liquid in a cylindrical container. A massless piston of area $A$ floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass $m$ is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, \[ \frac{dr}{r} \], is

\[ (1) \frac{K\alpha}{mg} \quad (2) \frac{K\alpha}{3mg} \quad (3) \frac{mg}{3Ka} \quad (4) \frac{mg}{Ka} \]

Answer (3)

Sol. $K = -\frac{dP}{dV}$

\[ -\frac{dV}{V} = \frac{dP}{K} = \frac{mg}{Ka} \]

\[ \Rightarrow \frac{dr}{r} = \frac{mg}{3Ka} \]

10. Two moles of an ideal monoatomic gas occupies a volume $V$ at $27^\circ C$. The gas expands adiabatically to a volume $2V$. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K (b) 2.7 kJ
(2) (a) 195 K (b) –2.7 kJ
(3) (a) 189 K (b) –2.7 kJ
(4) (a) 195 K (b) 2.7 kJ

Answer (3)

Sol. \[ TV^{\gamma - 1} = \text{Constant} \]

\[ T_f = 300 \left( \frac{V}{2V} \right)^{\gamma - 1} = 189 K \]

\[ \Delta U = nC_v \Delta T = 2 \times \frac{3R}{2} \times [189 - 300] = -2.7 \text{ kJ} \]

11. The mass of a hydrogen molecule is $3.32 \times 10^{-27}$ kg. If $10^{23}$ hydrogen molecules strike, per second, a fixed wall of area $2 \text{ cm}^2$ at an angle of $45^\circ$ to the normal, and rebound elastically with a speed of $10^3$ m/s, then the pressure on the wall is nearly

(1) $2.35 \times 10^3$ N/m$^2$
(2) $4.70 \times 10^3$ N/m$^2$
(3) $2.35 \times 10^2$ N/m$^2$
(4) $4.70 \times 10^2$ N/m$^2$

Answer (1)

Sol. \[ P = \frac{F}{A} = \frac{2nmv\cos\theta \times 2}{A} \]

\[ = \frac{2 \times 10^{23} \times 3.32 \times 10^{-27} \times 10^3}{\sqrt{2} \times 2 \times 10^{-4}} \text{N/m}^2 \]

\[ = 2.35 \times 10^3 \text{ N/m}^2 \]

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}$/second. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = $6.02 \times 10^{23}$ gm mole$^{-1}$)

(1) 6.4 N/m
(2) 7.1 N/m
(3) 2.2 N/m
(4) 5.5 N/m

Answer (2)
13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is \( 2.7 \times 10^3 \) kg/m³ and its Young’s modulus is \( 9.27 \times 10^{10} \) Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 5 kHz  
(2) 2.5 kHz  
(3) 10 kHz  
(4) 7.5 kHz

Answer (1)

Sol. \( f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} = 10^{12} \)

\[ f = \frac{1}{2\pi} \sqrt{\frac{4\pi^2 m \times 10^{24}}{6.02 \times 10^{23}}} \times 10^{24} \]

\[ K = 4\pi^2 m \times 10^{24} = \frac{4 \times 10 \times 108 \times 10^{-3}}{6.02 \times 10^{23}} \times 10^{24} \]

\[ = 7.1 \text{ N/m} \]

14. Three concentric metal shells \( A, B \) and \( C \) of respective radii \( a, b \) and \( c \) \((a < b < c)\) have surface charge densities \(+\sigma\), \(-\sigma\) and \(+\sigma\) respectively. The potential of shell \( B \) is

(1) \( \frac{\sigma}{\varepsilon_0} \left[ \frac{a^2 - b^2}{a} + c \right] \)

(2) \( \frac{\sigma}{\varepsilon_0} \left[ \frac{a^2 - b^2}{b} + c \right] \)

(3) \( \frac{\sigma}{\varepsilon_0} \left[ \frac{b^2 - c^2}{b} + a \right] \)

(4) \( \frac{\sigma}{\varepsilon_0} \left[ \frac{b^2 - c^2}{c} + a \right] \)

Answer (2)

Sol. \[ V_B = \left[ \frac{\sigma 4\pi a^2}{4\pi \varepsilon_0 a} - \frac{\sigma 4\pi b^2}{4\pi \varepsilon_0 b} + \frac{\sigma 4\pi c^2}{4\pi \varepsilon_0 c} \right] \]

\[ V_B = \frac{\sigma}{\varepsilon_0} \left[ \frac{a^2 - b^2}{b} + c \right] \]

15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant \( K = \frac{5}{3} \) is inserted between the plates, the magnitude of the induced charge will be

(1) 1.2 nC  
(2) 0.3 nC  
(3) 2.4 nC  
(4) 0.9 nC

Answer (1)

Sol. \[ Q = K' Q_0 \]

\[ Q_{\text{induced}} = Q \left( 1 - \frac{1}{K} \right) \]

\[ = \frac{5}{3} \times 90 \times 10^{-12} \times 20 \left( 1 - \frac{3}{5} \right) \]

\[ = 1.2 \text{ nC} \]

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by

\[ e = 100 \sin 30t \]

\[ i = 20 \sin \left( 30t - \frac{\pi}{4} \right) \]

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(1) 50, 10  
(2) \( \frac{1000}{\sqrt{2}} \), 10  
(3) \( \frac{50}{\sqrt{2}} \), 0  
(4) 50, 0

Answer (2)
Sol. \[ P_{av} = E_{rms} I_{rms} \cos \phi \]
\[ = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}} \]
\[ i_{\text{wattless}} = i_{rms} \sin \phi = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10 \]

17. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between
(1) 11.6 V and 11.7 V  
(2) 11.5 V and 11.6 V  
(3) 11.4 V and 11.5 V  
(4) 11.7 V and 11.8 V

Answer (2)

Sol.
\[ 12 - x - 10(x + y) = 0 \]
\[ \Rightarrow 12 = 11x + 10y \quad \text{...(i)} \]
\[ 13 = 10x + 12y \quad \text{...(ii)} \]
Solving \( x = \frac{7}{16} \) A, \( y = \frac{23}{32} \) A
\[ V = 10(x + y) = 11.56 \text{ V} \]

Alternatively:
\[ r_e = \frac{2}{3} \Omega, \quad R = 10 \Omega \]
\[ \frac{E_{eq}}{E_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} \Rightarrow E_{eq} = \frac{37}{3} \text{ V} \]
\[ V = \frac{E_{eq}}{R + r_{eq}} = 11.56 \text{ V} \]

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii \( r_e, r_p, r_\alpha \) respectively in a uniform magnetic field \( B \). The relation between \( r_e, r_p, r_\alpha \) is
(1) \( r_e > r_p = r_\alpha \)  
(2) \( r_e < r_p = r_\alpha \)  
(3) \( r_e < r_p < r_\alpha \)  
(4) \( r_e < r_\alpha < r_p \)

Answer (2)

Sol.
\[ r = \frac{\sqrt{2mk}}{qB} \]
\[ \frac{r_e}{r_p} = \frac{\sqrt{2m_e}}{q_e} \quad \frac{q_e}{\sqrt{2m_p}} \quad [m_\alpha = 4m_p] \]
\[ \frac{r_\alpha}{q_\alpha} = 2q_p \]
\[ = 1 \]

Mass of electron is least and charge \( q_e = e \)
So, \( r_e < r_p = r_\alpha \)

19. The dipole moment of a circular loop carrying a current \( I \), is \( m \) and the magnetic field at the centre of the loop is \( B_1 \). When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is \( B_2 \). The ratio \( \frac{B_1}{B_2} \) is
(1) 2  
(2) \( \sqrt{3} \)  
(3) \( \frac{1}{\sqrt{2}} \)  
(4) \( \frac{1}{2} \)

Answer (3)

Sol.
\[ m = I(\pi R^2), \quad m' = 2m = I \times \left( \pi \sqrt{2} R \right)^2 \]
\[ B_1 = \frac{\mu_0 I}{2R} \]
\[ B_2 = \frac{\mu_0 I}{2 \times \left( \sqrt{2} R \right)} \]
\[ \therefore \frac{B_1}{B_2} = \sqrt{2} \]

20. For an RLC circuit driven with voltage of amplitude \( v_m \) and frequency \( \omega_0 = \frac{1}{\sqrt{LC}} \), the current exhibits resonance. The quality factor, \( Q \) is given by
(1) \( \frac{\omega_0 L}{R} \)  
(2) \( \frac{\omega_0 R}{L} \)  
(3) \( \frac{R}{(\omega_0 C)} \)  
(4) \( \frac{CR}{\omega_0} \)

Answer (1)

Sol. Quality factor, \( Q = \frac{\omega_0 L}{2(\Delta \omega)} \)
\[ Q = \frac{\omega_0 L}{R} \]
21. An EM wave from air enters a medium. The electric fields are \( \vec{E}_1 = E_{01} \hat{x} \cos \left[ 2\pi \left( \frac{z}{c} - t \right) \right] \) in air and \( \vec{E}_2 = E_{02} \hat{x} \cos[k(2z - ct)] \) in medium, where the wave number \( k \) and frequency \( \nu \) refer to their values in air. The medium is non-magnetic. If \( \varepsilon_1 \) and \( \varepsilon_2 \) refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) \( \frac{\varepsilon_1}{\varepsilon_2} = 4 \)  
(2) \( \frac{\varepsilon_1}{\varepsilon_2} = 2 \)  
(3) \( \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{4} \)  
(4) \( \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{2} \)

Answer (3)

Sol. \( \vec{E}_1 = E_{01} \hat{x} \cos \left[ 2\pi \left( \frac{z}{c} - t \right) \right] \) in air and \( \vec{E}_2 = E_{02} \hat{x} \cos[k(2z - ct)] \) in medium.

During refraction, frequency remains unchanged, whereas wavelength gets changed.

\[ \therefore k' = 2k \]  
(From equations)

\[ \Rightarrow \frac{2\pi}{\lambda'} = 2 \left( \frac{2\pi}{\lambda_0} \right) \]

\[ \Rightarrow \lambda' = \frac{\lambda_0}{2} \]

\[ \Rightarrow \nu = \frac{c}{2} \]

\[ \Rightarrow \frac{1}{\sqrt{\mu_0 \varepsilon_2}} = \frac{1}{2} \times \frac{1}{\sqrt{\mu_0 \varepsilon_1}} \]

\[ \Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{4} \]

22. Unpolarized light of intensity \( I \) passes through an ideal polarizer \( A \). Another identical polarizer \( B \) is placed behind \( A \). The intensity of light beyond \( B \) is found to be \( \frac{I}{2} \). Now another identical polarizer \( C \) is placed between \( A \) and \( B \). The intensity beyond \( B \) is now found to be \( \frac{I}{8} \). The angle between polarizer \( A \) and \( C \) is

(1) 0°  
(2) 30°  
(3) 45°  
(4) 60°

Answer (3)

Sol. Polaroids \( A \) and \( B \) are oriented with parallel pass axis

Let polaroid \( C \) is at angle \( \theta \) with \( A \) then it makes \( \theta \) with \( B \) also.

\[ \therefore \frac{I}{8} = \left( \frac{1}{2} \times \cos^2 \theta \right) \cos^2 \theta \]

\[ \Rightarrow \cos^2 \theta = \frac{1}{2} \]

\[ \Rightarrow \theta = 45° \]

23. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young’s fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?

(\( i.e. \) distance between the centres of each slit.)

(1) 25 μm  
(2) 50 μm  
(3) 75 μm  
(4) 100 μm

Answer (1)

Sol. \( \text{dsin}\theta = \lambda \)

\[ \lambda = \frac{d}{2} \quad [d = 1 \times 10^{-6} \text{ m}] \]

\[ \Rightarrow \lambda = 5000 \text{ Å} \]

Fringe width, \( B = \frac{\lambda D}{d'} \) (\( d' \) is slit separation)

\[ 10^{-2} = \frac{5000 \times 10^{-10} \times 0.5}{d'} \]

\[ \Rightarrow d' = 25 \times 10^{-6} \text{ m} = 25 \text{ μm} \]
24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let \( \lambda_n, \lambda_g \) be the de Broglie wavelength of the electron in the \( n^{th} \) state and the ground state respectively. Let \( \Lambda_n \) be the wavelength of the emitted photon in the transition from the \( n^{th} \) state to the ground state. For large \( n \), \( (A, B \) are constants)

\[
\begin{align*}
(1) \quad \Lambda_n &= A + \frac{B}{\lambda_n^2} \\
(2) \quad \Lambda_n &= A + B\lambda_n \\
(3) \quad \Lambda_n^2 &= A + B\lambda_n^2 \\
(4) \quad \Lambda_n^2 &= \lambda \\
\end{align*}
\]

Answer (1)

Sol. \( P_n = \frac{h}{\lambda_n}, P_g = \frac{h}{\lambda_g} \)

\[
\begin{align*}
k &= P^2 = \frac{h^2}{2m} \quad E = -k = -\frac{h^2}{2m\lambda_n^2} \\
E_n &= \frac{h^2}{2m\lambda_n^2} \quad E_g = \frac{h^2}{2m\lambda_g^2} \\
E_n - E_g &= \frac{h^2}{2m} \left( \frac{1}{\lambda_n^2} - \frac{1}{\lambda_g^2} \right) = \frac{hc}{\Lambda_n} \\
\end{align*}
\]

\[
\begin{align*}
\frac{h^2}{2m} \left( \lambda_n^2 - \lambda_g^2 \right) &= \frac{hc}{\Lambda_n} \\
\Lambda_n &= \frac{2mc\lambda_n^2}{h} \left( \frac{\lambda_n^2}{\lambda_n^2 - \lambda_g^2} \right) \\
\end{align*}
\]

\[
\begin{align*}
\Delta E &= \frac{2mc\lambda_n^2}{h} \left[ \frac{\lambda_n^2}{\lambda_n^2 - \lambda_g^2} \right]^{-1} \\
&= \frac{2mc\lambda_n^2}{h} \left[ 1 - \frac{\lambda_g^2}{\lambda_n^2} \right]^{-1} \\
&= \frac{2mc\lambda_n^2}{h} \left[ 1 + \frac{\lambda_g^2}{\lambda_n^2} \right] \\
\end{align*}
\]

25. If the series limit frequency of the Lyman series is \( \nu_L \), then the series limit frequency of the Pfund series is

\[
\begin{align*}
(1) & \quad 25 \nu_L \\
(2) & \quad 16 \nu_L \\
(3) & \quad \nu_L/16 \\
(4) & \quad \nu_L/25 \\
\end{align*}
\]

Answer (4)

Sol. \( \nu_L = E \left[ \frac{1}{12} - \frac{1}{\infty} \right] = E \)

\[
\begin{align*}
\nu_P &= E \left[ \frac{1}{25} \right] = E \\
\Rightarrow \quad \nu_P &= \frac{\nu_L}{25} \\
\end{align*}
\]

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is \( p_d \); while for its similar collision with carbon nucleus at rest, fractional loss of energy is \( p_c \). The values of \( p_d \) and \( p_c \) are respectively

\[
\begin{align*}
(1) & \quad (.89, .28) \\
(2) & \quad (.28, .89) \\
(3) & \quad (0, 0) \\
(4) & \quad (0, 1) \\
\end{align*}
\]

Answer (1)

Sol. \( m\nu = mv_1 + 2m \times v_2 \) \( \cdots \)(i)

\[
\begin{align*}
u &= (v_2 - v_1) \quad \cdots \&&(\text{ii}) \\
\Rightarrow \quad v_1 &= -\frac{u}{3} \\
\end{align*}
\]

\[
\begin{align*}
\frac{\Delta E}{E} &= p_d = \frac{1}{2} \left( \frac{mv_1^2}{3} \right)^2 \\
&= \frac{8}{9} = 0.89 \\
\end{align*}
\]
29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 \( \Omega \), a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1 \( \Omega \)  
(2) 1.5 \( \Omega \)  
(3) 2 \( \Omega \)  
(4) 2.5 \( \Omega \)

Answer (2)

Sol. \[ E \propto l \]
and \[ E = ir \propto l \]
\[ \therefore \frac{E}{E - ir} = \frac{l_1}{l_2} \]
\[ \Rightarrow \frac{E}{E - \left( \frac{E}{r + 5} \right) \times r} = \frac{52}{40} \]
\[ \Rightarrow \frac{r + 5}{r} = \frac{13}{10} \]
\[ \Rightarrow r = 1.5 \Omega \]

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 k\( \Omega \). How much was the resistance on the left slot before interchanging the resistances?

(1) 990 \( \Omega \)  
(2) 505 \( \Omega \)  
(3) 550 \( \Omega \)  
(4) 910 \( \Omega \)

Answer (3)

Sol. \[ R_1 + R_2 = 1000 \Omega \]
\[ R_1 = 550 \Omega \]
31. The ratio of mass percent of C and H of an organic compound \( (C_xH_yO_z) \) is 6 : 1. If one molecule of the above compound \( (C_xH_yO_z) \) contains half as much oxygen as required to burn one molecule of compound \( C_xH_y \) completely to \( CO_2 \) and \( H_2O \). The empirical formula of compound \( C_xH_yO_z \) is

(1) \( C_3H_6O_3 \)  
(2) \( C_2H_4O \)  
(3) \( C_3H_4O_2 \)  
(4) \( C_2H_4O_3 \)

Answer (4)

Sol.  
<table>
<thead>
<tr>
<th>Element</th>
<th>Relative mass</th>
<th>Relative mole</th>
<th>Simplest whole number ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6</td>
<td>( \frac{6}{12} = 0.5 )</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>( \frac{1}{1} = 1 )</td>
<td>2</td>
</tr>
</tbody>
</table>

So, \( X = 1, Y = 2 \)

Equation for combustion of \( C_xH_y \)

\[
C_xH_y + \left( X + \frac{Y}{4} \right)O_2 \rightarrow XCO_2 + \frac{Y}{2}H_2O
\]

Oxygen atoms required \( = 2 \left( X + \frac{Y}{4} \right) \)

As per information,

\[
2 \left( X + \frac{Y}{4} \right) = 2Z
\]

\[
\Rightarrow \left( 1 + \frac{2}{4} \right) = Z
\]

\[
\Rightarrow Z = 1.5
\]

Molecule can be written

\[
C_xH_yO_z \\
C_1H_2O_{3/2}
\]

\[
\Rightarrow C_2H_4O_3
\]

33. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) \( \text{He}_2^+ \)  
(2) \( \text{He}_2^- \)  
(3) \( \text{H}_2^- \)  
(4) \( \text{H}_2^0 \)

Answer (4)

Sol.  

\[
\text{Electronic configuration} \quad \text{Bond order}
\]

\[
\begin{align*}
\text{He}_2^+ & : \sigma_{1s}^2 \sigma_{1s}^+ \\
\text{He}_2^- & : \sigma_{1s}^2 \sigma_{1s}^- \\
\text{H}_2^- & : \sigma_{1s}^2 \sigma_{1s}^2 \\
\text{H}_2^0 & : \sigma_{1s}^2
\end{align*}
\]

Molecule having zero bond order will not be a viable molecule.

34. Which of the following lines correctly show the temperature dependence of equilibrium constant \( K \), for an exothermic reaction?

(1) A and B  
(2) B and C  
(3) C and D  
(4) A and D

Answer (1)

Sol.  

Equilibrium constant \( K = \left( \frac{A_f}{A_b} \right) e^{\frac{-\Delta H^o}{RT}} \)

\[
\ln K = \ln \left( \frac{A_f}{A_b} \right) - \frac{\Delta H^o}{RT} \left( \frac{1}{T} \right)
\]

Comparing with equation of straight line,

\[
Slope = \frac{-\Delta H^o}{R}
\]

Since, reaction is exothermic, \( \Delta H^o = -ve \), therefore, slope = +ve.
37. An aqueous solution contains 0.10 M H$_2$S and 0.20 M HCl. If the equilibrium constant for the formation of HS$^-$ from H$_2$S is $1.0 \times 10^{-7}$ and that of S$^{2-}$ from HS$^-$ ions is $1.2 \times 10^{-13}$ then the concentration of S$^{2-}$ ions in aqueous solution is

(1) $5 \times 10^{-8}$
(2) $3 \times 10^{-20}$
(3) $6 \times 10^{-21}$
(4) $5 \times 10^{-19}$

Answer (2)

Sol. In presence of external H$^+$,

\[
\text{H}_2\text{S} \rightleftharpoons 2\text{H}^+ + \text{S}^{2-}, \quad K_a_1 \cdot K_a_2 = K_{eq}
\]

\[
\frac{[\text{H}^+]^2 [\text{S}^{2-}]}{[\text{H}_2\text{S}]} = 1 \times 10^{-7} \times 1.2 \times 10^{-13}
\]

\[
[\text{S}^{2-}] = 3 \times 10^{-20}
\]

38. An aqueous solution contains an unknown concentration of Ba$^{2+}$. When 50 mL of a 1 M solution of Na$_2$SO$_4$ is added, BaSO$_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of BaSO$_4$ is $1 \times 10^{-10}$. What is original concentration of Ba$^{2+}$?

(1) $5 \times 10^{-9}$ M
(2) $2 \times 10^{-9}$ M
(3) $1.1 \times 10^{-9}$ M
(4) $1.0 \times 10^{-10}$ M

Answer (3)

Sol. Final concentration of [SO$_{4}^{2-}$] = \[
\frac{[50 \times 1]}{[500]} = 0.1 \text{ M}
\]

$K_{sp}$ of BaSO$_4$,

\[
[\text{Ba}^{2+}][\text{SO}_4^{2-}] = 1 \times 10^{-10}
\]

\[
[\text{Ba}^{2+}][0.1] = \frac{10^{-10}}{0.1} = 10^{-9} \text{ M}
\]

Concentration of Ba$^{2+}$ in final solution = $10^{-9}$ M

Concentration of Ba$^{2+}$ in the original solution.

\[
M_1V_1 = M_2V_2
\]

\[
M_1 (500 - 50) = 10^{-9} (500)
\]

\[
M_1 = 1.11 \times 10^{-9} \text{ M}
\]

So, option (3) is correct.
39. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 torr, was 1.00 torr s⁻¹ when 5% had reacted and 0.5 torr s⁻¹ when 33% had reacted. The order of the reaction is

(1) 2
(2) 3
(3) 1
(4) 0

Answer (1)

Sol. Assume the order of reaction with respect to acetaldehyde is x.

**Condition-1:**

\[ \text{Rate} = k[CH}_3\text{CHO]}^x \]

\[ 1 = k[363 \times 0.95]^x \]

\[ 1 = k[344.85]^x \]  \( \text{...(i)} \)

**Condition-2:**

\[ 0.5 = k[363 \times 0.67]^x \]

\[ 0.5 = k[243.21]^x \]  \( \text{...(ii)} \)

Divide equation (i) by (ii),

\[ \frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \]

\[ \Rightarrow 2 = (1.414)^x \]

\[ \Rightarrow x = 2 \]

40. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

(1) 6.4 hours  \( \quad \) (2) 0.8 hours  \( \quad \) (3) 3.2 hours  \( \quad \) (4) 1.6 hours

Answer (3)

**Sol.** B₂H₆ + 3O₂ → B₂O₃ + 3H₂O

27.66 g of B₂H₆ = 1 mole of B₂H₆ which requires three moles of oxygen (O₂) for complete burning

6H₂O → 6H₂⁺ + 3O₂ (On electrolysis)

Number of faradays = 12 = Amount of charge

\[ 12 \times 96500 = i \times t \]

\[ 12 \times 96500 = 100 \times t \]

\[ t = \frac{12 \times 96500}{100} \text{ second} \]

\[ t = \frac{12 \times 96500}{100 \times 3600} \text{ hour} \]

\[ t = 3.2 \text{ hours} \]

41. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting [3Ca₃(PO₄)₂.Ca(OH)₂] to

(1) [CaF₂]  \( \quad \) (2) [3(CaF₂).Ca(OH)₂]

(3) [3Ca₃(PO₄)₂.CaF₂]  \( \quad \) (4) [3(Ca(OH)₂).CaF₂]

Answer (3)

Sol. F⁻ ions make the teeth enamel harder by converting

[3Ca₃(PO₄)₂.Ca(OH)₂] to [3Ca₃(PO₄)₂.CaF₂]

42. Which of the following compounds contain(s) no covalent bond(s)?

KCl, PH₃, O₂, B₂H₆, H₂SO₄

(1) KCl, B₂H₆  \( \quad \) (2) KCl, H₂SO₄

(3) KCl  \( \quad \) (4) KCl, B₂H₆

Answer (3)

Sol. KCl – Ionic bond between K⁺ and Cl⁻

PH₃ – Covalent bond between P and H

O₂ – Covalent bond between O atoms

B₂H₆ – Covalent bond between B and H atoms

H₂SO₄ – Covalent bond between S and O and also between O and H.

∴ Compound having no covalent bonds is KCl only.

43. Which of the following are Lewis acids?

(1) PH₃ and BCl₃  \( \quad \) (2) AlCl₃ and SiCl₄

(3) PH₃ and SiCl₄  \( \quad \) (4) BCl₃ and AlCl₃

Answer (4)*

Sol. BCl₃ – electron deficient, incomplete octet

AlCl₃ – electron deficient, incomplete octet

Ans-(4) BCl₃ and AlCl₃

SiCl₄ can accept lone pair of electron in d-orbital of silicon hence it can act as Lewis acid.

* Although the most suitable answer is (4). However, both option (4) & (2) can be considered as correct answers.

e.g. hydrolysis of SiCl₄

Hence option (2), AlCl₃ and SiCl₄ is also correct.
44. Total number of lone pair of electrons in $\text{I}_3^-$ ion is
   (1) 3
   (2) 6
   (3) 9
   (4) 12
   Answer (3)
   Sol. Structure of $\text{I}_3^-$

   Number of lone pairs in $\text{I}_3^-$ is 9.

45. Which of the following salts is the most basic in aqueous solution?
   (1) Al(CN)$_3$
   (2) CH$_3$COOK
   (3) FeCl$_3$
   (4) Pb(CH$_3$COO)$_2$
   Answer (2)
   Sol. CH$_3$COOK + H$_2$O $\rightarrow$ CH$_3$COOH + KOH

   FeCl$_3$ – Acidic solution
   Al(CN)$_3$ – Salt of weak acid and weak base
   Pb(CH$_3$COO)$_2$ – Salt of weak acid and weak base
   CH$_3$COOK is salt of weak acid and strong base.
   Hence solution of CH$_3$COOK is basic.

46. Hydrogen peroxide oxidises $[\text{Fe(CN)}_6]^{4-}$ to $[\text{Fe(CN)}_6]^{3-}$ in acidic medium but reduces $[\text{Fe(CN)}_6]^{3-}$ to $[\text{Fe(CN)}_6]^{4-}$ in alkaline medium. The other products formed are, respectively.
   (1) (H$_2$O + O$_2$) and H$_2$O
   (2) (H$_2$O + O$_2$) and (H$_2$O + OH$^-$)
   (3) H$_2$O and (H$_2$O + O$_2$)
   (4) H$_2$O and (H$_2$O + OH$^-$)
   Answer (3)
   Sol. $[\text{Fe(CN)}_6]^{3-} + 1/2 \text{H}_2\text{O}_2 + \text{H}^+ \rightarrow [\text{Fe(CN)}_6]^{4-} + \text{H}_2\text{O}$

47. The oxidation states of Cr in $[\text{Cr(H}_2\text{O)}_6]^{3+}$, $[\text{Cr(C}_6\text{H}_6)_2]^{3+}$ and $K_2[\text{Cr(CN)}_2(O_2)(\text{O}_2)(\text{NH}_3)]$ respectively are
   (1) +3, +4 and +6
   (2) +3, +2 and +4
   (3) +3, 0 and +6
   (4) +3, 0 and +4
   Answer (3)
   Sol. $[\text{Cr(H}_2\text{O)}_6]^{3+}$ $\Rightarrow x + 0 \times 6 - 1 \times 3 = 0$

   $[\text{Cr(C}_6\text{H}_6)_2]^{3+}$ $\Rightarrow x + 2 \times 0 = 0$

   $K_2[\text{Cr(CN)}_2(O_2)(\text{O}_2)(\text{NH}_3)]$

   $\Rightarrow 1 \times 2 + x - 1 \times 2 - 2 \times 2 - 2 \times 1 = 0$

   $\Rightarrow x = 6$

48. The compound that does not produce nitrogen gas by the thermal decomposition is
   (1) Ba(N$_3$)$_2$
   (2) (NH$_4$)$_2$Cr$_2$O$_7$
   (3) NH$_4$NO$_2$
   (4) (NH$_4$)$_2$SO$_4$
   Answer (4)
   Sol. $\text{(NH}_4)^2\text{Cr}_2\text{O}_7 \xrightarrow{\Delta} \text{N}_2 + 4\text{H}_2\text{O} + \text{Cr}_2\text{O}_3$

   $\text{NH}_4\text{NO}_2 \xrightarrow{\Delta} \text{N}_2 + 2\text{H}_2\text{O}$

   $\text{(NH}_4)^2\text{SO}_4 \xrightarrow{\Delta} 2\text{NH}_3 + \text{H}_2\text{SO}_4$

   $\text{Ba(N}_3)_2 \xrightarrow{\Delta} \text{Ba} + 3\text{N}_2$

   Among all the given compounds, only (NH$_4$)$_2$SO$_4$ do not form dinitrogen on heating, it produces ammonia gas.

49. When metal ‘M’ is treated with NaOH, a white gelatinous precipitate ‘X’ is obtained, which is soluble in excess of NaOH. Compound ‘X’ when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal ‘M’ is
   (1) Zn
   (2) Ca
   (3) Al
   (4) Fe
Sol. \[
\begin{align*}
\text{Al}^{3+} + \text{NaOH} & \rightarrow \text{Al(OH)}_3 \downarrow \quad \text{White gelatinous ppt.} \\
\text{Excess NaOH} & \rightarrow \text{NaAlO}_2 \\
2\text{Al(OH)}_3 \quad \text{Strong heating} & \rightarrow \text{Al}_2\text{O}_3 + 3\text{H}_2\text{O} \\
\text{Al}_2\text{O}_3 & \text{is used in column chromatography.}
\end{align*}
\]

50. Consider the following reaction and statements

\[ [\text{Co(NH}_3\text{)}_4\text{Br}_2]^+ + \text{Br}^- \rightarrow [\text{Co(NH}_3\text{)}_3\text{Br}_3] + \text{NH}_3 \]

(I) Two isomers are produced if the reactant complex ion is a cis-isomer

(II) Two isomers are produced if the reactant complex ion is a trans-isomer.

(III) Only one isomer is produced if the reactant complex ion is a trans-isomer.

(IV) Only one isomer is produced if the reactant complex ion is a cis-isomer.

The correct statements are:

(1) (I) and (II) 

(2) (I) and (III) 

(3) (III) and (IV) 

(4) (II) and (IV)

Answer (2)

Sol.

\[
\begin{align*}
\text{CHO} & \quad \text{CH}_2\text{-OH} \\
\text{CH} & \equiv \text{C} \rightarrow \text{CH}_3\text{-C} \equiv \text{CH}_3 \\
& \text{Na/liq. NH}_3 \rightarrow \text{H} \quad \text{H} \\
& \text{C} = \text{C} \\
& \text{H} \\
\end{align*}
\]

So option (2) is correct.

51. Glucose on prolonged heating with HI gives

(1) \( n \)-Hexane

(2) 1-Hexene

(3) Hexanoic acid

(4) 6-iodohexanal

Answer (1)

Sol.

\[
\begin{align*}
\text{CHO} & \quad \text{CH}_2\text{-OH} \\
\text{CH} & \equiv \text{C} \rightarrow \text{CH}_3\text{-C} \equiv \text{CH}_3 \\
& \text{Na/liq. NH}_3 \rightarrow \text{H} \quad \text{H} \\
& \text{C} = \text{C} \\
& \text{H} \\
\end{align*}
\]

So option (2) is correct.

52. The trans-alkenes are formed by the reduction of alkynes with

(1) \( \text{H}_2 - \text{Pd/C, BaSO}_4 \)

(2) \( \text{NaBH}_4 \)

(3) \( \text{Na/liq. NH}_3 \)

(4) \( \text{Sn - HCl} \)

Answer (3)

Sol.

\[
\begin{align*}
\text{CH}_3\text{-C} & \equiv \text{C} \equiv \text{CH}_3 \rightarrow \text{CH}_3\text{-C} \equiv \text{CH}_3 \\
& \text{Na/liq. NH}_3 \\
\end{align*}
\]

So, option (3) is correct.

53. Which of the following compounds will be suitable for Kjeldahl’s method for nitrogen estimation?

(1) \[
\begin{align*}
\text{NH}_2
\end{align*}
\]

(2) \[
\begin{align*}
\text{NH}_2
\end{align*}
\]

(3) \[
\begin{align*}
\text{NH}_2
\end{align*}
\]

(4) \[
\begin{align*}
\text{NH}_2
\end{align*}
\]

Answer (2)
Sol. Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl’s method.

54. Phenol on treatment with CO\(_2\) in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with (CH\(_3\)CO)\(_2\)O in the presence of catalytic amount of H\(_2\)SO\(_4\) produces

![Chemical Structures]

Answer (1)

55. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

<table>
<thead>
<tr>
<th>Base</th>
<th>Acid</th>
<th>End point</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Weak</td>
<td>Strong</td>
<td>Colourless to pink</td>
</tr>
<tr>
<td>(2) Strong</td>
<td>Strong</td>
<td>Pinkish red to yellow</td>
</tr>
<tr>
<td>(3) Weak</td>
<td>Strong</td>
<td>Yellow to pinkish red</td>
</tr>
<tr>
<td>(4) Strong</td>
<td>Strong</td>
<td>Pink to colourless</td>
</tr>
</tbody>
</table>

Answer (3)

Sol. The pH range of methyl orange is

\[\text{Pinkish red} \quad 3.9 \quad 4.5 \quad \text{Yellow}\]

Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

56. The predominant form of histamine present in human blood is (pK\(_a\), Histidine = 6.0)

![Chemical Structures]

Answer (4)

Sol. Histamine

At pH (7.4) major form of histamine is protonated at primary amine.

57. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br\(_2\) to form product B. A and B are respectively

![Chemical Structures]
58. The increasing order of basicity of the following compound is
   (a) < (b) < (c) < (d)  (2) (b) < (a) < (c) < (d)  (3) (b) < (a) < (d) < (c)  (4) (d) < (b) < (a) < (c)

   Answer (3)

   Sol. (a) \[ \text{Protonation} \rightarrow \text{NH}^+ \text{NH}_3^- \]
   (b) \[ \text{Protonation} \rightarrow \text{NH}_2 \text{sp}^2 \]
   (c) \[ \text{Protonation} \rightarrow \text{NH}_2 \text{sp}^3 \]
   (d) \[ \text{Protonation} \rightarrow \text{NH}_2^- \text{CH}_3 \]

   \[ \therefore \text{Correct order of basicity} : b < a < d < c. \]

59. The major product formed in the following reaction is

   Answer (4)

   Sol. \[ \text{H}_2\text{O} \rightarrow \text{H}^+ \text{OH}^- \]

   Hence, option (4) is correct.

60. The major product of the following reaction is

   Answer (2)

   Sol. \[ \text{CH}_2\text{O}^- \text{is a strong base and strong nucleophile, so favourable condition is S}_\text{N}2/\text{E2}. \]

   Given alkyl halide is 2° and \( \beta \) C’s are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over S\(_\text{N}2\).

   Also, polarity of CH\(_3\)OH (solvent) is not as high as H\(_2\)O, so E1 is also dominated by E2.
61. Two sets $A$ and $B$ are as under:

$A = \{(a, b) \in \mathbb{R} \times \mathbb{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$

$B = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$

then

1. $B \subseteq A$
2. $A \subseteq B$
3. $A \cap B = \emptyset$ (an empty set)
4. Neither $A \subseteq B$ nor $B \subseteq A$

Answer (2)

Sol. As, $|a - 5| < 1 \text{ and } |b - 5| < 1$

$\Rightarrow 4 < a, b < 6$ and $\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \leq 1$

Taking axes as $a$-axis and $b$-axis

The set $A$ represents square $PQRS$ inside set $B$ representing ellipse and hence $A \subseteq B$.

62. Let $S = \{x \in \mathbb{R} : x \geq 0 \text{ and } x^2 - x + 1 = 0\}$. Then $S$:

1. Is an empty set
2. Contains exactly one element
3. Contains exactly two elements
4. Contains exactly four elements

Answer (3)

Sol. $2|\sqrt{x} - 3| + \sqrt{x}(|\sqrt{x} - 6| + 6) = 0$. Then $S$:

$x = -4$ makes all three row identical hence $(x + 4)^2$ will be factor

Also, $C_1 \rightarrow C_1 + C_2 + C_2$

$5x - 4 \quad 2x \quad 2x
\Delta = 5x - 4 \quad x - 4 \quad 2x
5x - 4 \quad 2x \quad x - 4
\Rightarrow 5x - 4$ is a factor

$\Delta = \lambda(5x - 4)(x + 4)^2$

$\therefore B = 5, A = -4$
65. If the system of linear equations
\[ \begin{align*}
    x + ky + 3z &= 0 \\
    3x + ky - 2z &= 0 \\
    2x + 4y - 3z &= 0
\end{align*} \]
has a non-zero solution \((x, y, z)\), then \(\frac{xyz}{y^2}\) is equal to
(1) \(-10\)  
(2) \(10\)  
(3) \(-30\)  
(4) \(30\)

Answer (2)

Sol. \(\because\) System of equation has non-zero solution.
\[\therefore \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0\]
\[\Rightarrow 44 - 4k = 0\]
\[\therefore k = 11\]
Let \(z = \lambda\).
\[\therefore x + 11y = -3\lambda\]
and \(3x + 11y = 2\lambda\)
\[\therefore x = \frac{5\lambda}{2}, \ y = \frac{-\lambda}{2}, \ z = \lambda\]
\[\therefore \frac{xyz}{y^2} = \frac{5\lambda \cdot \lambda}{2 \left(-\frac{\lambda}{2}\right)^2} = 10\]

66. From 6 different novels and 3 different dictionaries,
4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is
(1) At least 1000  
(2) Less than 500  
(3) At least 500 but less than 750  
(4) At least 750 but less than 1000

Answer (1)

Sol. Number of ways of selecting 4 novels from 6 novels = \(^6C_4\)
Number of ways of selecting 1 dictionary from 3 dictionaries = \(^3C_1\)
Required arrangements = \(^6C_4 \times ^3C_1 \times 4! = 1080\)
\[\Rightarrow\text{Atleast 1000}\]

67. The sum of the co-efficients of all odd degree terms
in the expansion of \(\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5\),
\((x > 1)\) is
(1) \(-1\)  
(2) \(0\)  
(3) \(1\)  
(4) \(2\)

Answer (4)

Sol. \(\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5\)
\[= 2\left[^5C_0 x^5 + ^5C_2 x^3 (x^3 - 1) + ^5C_4 x(x^3 - 1)^2\right]\]
\[= 2\left[x^5 + 10(x^6 - x^3) + 5x(x^6 - 2x^3 + 1)\right]\]
\[= 2\left[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x\right]\]
\[= 2\left[5x^7 + 10x^8 + x^5 - 10x^4 + 10x^3 + 5x\right]\]
Sum of odd degree terms coefficients
\[= 2(5 + 1 - 10 + 5)\]
\[= 2\]

68. Let \(a_1, a_2, a_3, \ldots, a_{49}\) be in A.P. such that
\[\sum_{k=0}^{12} a_{4k+1} = 416\] and \(a_9 + a_{43} = 66\).
If \(a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m\), then \(m\) is equal to
(1) \(66\)  
(2) \(68\)  
(3) \(34\)  
(4) \(33\)

Answer (3)

Sol. Let \(a_1 = a\) and common difference = \(d\)
Given, \(a_1 + a_5 + a_9 + \ldots + a_{49} = 416\)
\[\Rightarrow a + 24d = 32\] \((i)\)
Also, \(a_9 + a_{43} = 66 \Rightarrow a + 25d = 33\) \((ii)\)
Solving (i) & (ii),
We get \(d = 1, \ a = 8\)
Now, \(a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m\)
\[\Rightarrow 8^2 + 9^2 + \ldots + 24^2 = 140m\]
\[\Rightarrow \frac{24 \times 25 \times 49 - 7 \times 8 \times 15}{6} = 140m\]
\[\Rightarrow m = 34\]
69. Let $A$ be the sum of the first 20 terms and $B$ be the sum of the first 40 terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$$

If $B - 2A = 100\lambda$, then $\lambda$ is equal to

1. 232 2. 248 3. 464 4. 496

Answer (2)

Sol. 

$$A = 1^2 + 2^2 + 3^2 + \ldots + 20^2$$

$$= \frac{20 \times 21 \times 41}{6} + \frac{4 \times 10 \times 11 \times 21}{6}$$

$$= 2870 + 1540 = 4410$$

$$B = 1^2 + 2^2 + 3^2 + \ldots + 40^2$$

$$= \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6}$$

$$= 22140 + 11480 = 33620$$

$$\Rightarrow B - 2A = 33620 - 8820 = 24800$$

$$\Rightarrow 100\lambda = 24800$$

$$\lambda = 248$$

70. For each $t \in \mathbb{R}$, let $[t]$ be the greatest integer less than or equal to $t$. Then

$$\lim_{x \to 0^+} x \left( \left[ \frac{1}{x} \right] + \left[ \frac{2}{x} \right] + \ldots + \left[ \frac{15}{x} \right] \right)$$

1. Is equal to 0 2. Is equal to 15 3. Is equal to 120 4. Does not exist (in $\mathbb{R}$)

Answer (3)

Sol. 

$$\frac{1}{x} - 1 < \left[ \frac{1}{x} \right] \leq \frac{1}{x}$$

$$\frac{2}{x} - 1 < \left[ \frac{2}{x} \right] \leq \frac{2}{x}$$

$$\frac{15}{x} - 1 < \left[ \frac{15}{x} \right] \leq \frac{15}{x}$$

$$\sum_{r=1}^{15} \left( \frac{r}{x} - 1 \right) < \sum_{r=1}^{15} \left[ \frac{r}{x} \right] \leq \sum_{r=1}^{15} \frac{r}{x}$$

$$120 < \lim_{x \to 0^+} x \left( \sum_{r=1}^{15} \left[ \frac{r}{x} \right] \right) \leq 120$$

$$\Rightarrow \lim_{x \to 0^+} x \left( \left[ \frac{1}{x} \right] + \left[ \frac{2}{x} \right] + \ldots + \left[ \frac{15}{x} \right] \right) = 120$$

71. Let $S = \{t \in \mathbb{R} : f(x) = |x - \pi| (e^{|x|} - 1) \sin x |x| \text{ is not differentiable at } t\}$. Then the set $S$ is equal to

1. $\phi$ (an empty set) 2. $\{0\}$ 3. $\{\pi\}$ 4. $\{0, \pi\}$

Answer (1)

Sol. 

$$f(x) = |x - \pi| (e^{|x|} - 1) \sin x |x|$$

$x = \pi$, $0$ are repeated roots and also continuous. Hence, $f$ is differentiable at all $x$.

72. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of $b$ is

1. 6 2. $\frac{7}{2}$ 3. 4 4. $\frac{9}{2}$

Answer (4)

Sol. 

$$y^2 = 6x; \text{ slope of tangent at } (x_1, y_1) \text{ is } m_1 = \frac{3}{y_1}$$

also $9x^2 + by^2 = 16; \text{ slope of tangent at } (x_1, y_1) \text{ is } m_2 = \frac{-9x_1}{by_1}$

As $m_1 m_2 = -1$

$$\Rightarrow \frac{-27x_1}{by_1} = -1$$

$$\Rightarrow b = \frac{9}{2} \text{ (as } y_1^2 = 6x_1)$$

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}, x \in \mathbb{R} - \{-1, 0, 1\}$.

If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

1. 3 2. $-3$ 3. $-2\sqrt{2}$ 4. $2\sqrt{2}$

Answer (4)

Sol. 

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}}$$

$$= \left( x - \frac{1}{x} \right) + \frac{2}{x - \frac{1}{x}}$$
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74. The integral
\[ \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)(\sin^3 x + \cos^3 x)} \, dx \]
is equal to
(1) \( \frac{1}{3(1 + \tan^3 x)} + C \) 
(2) \( \frac{-1}{3(1 + \tan^3 x)} + C \) 
(3) \( \frac{1}{1 + \cot^3 x} + C \) 
(4) \( \frac{-1}{1 + \cot^3 x} + C \) 
(where \( C \) is a constant of integration)

Answer (2)

Sol. 
\[ I = \int \frac{\sin^2 x \cdot \cos^2 x}{(\sin^3 x + \cos^3 x)(\sin^3 x + \cos^3 x)} \, dx \]

Dividing the numerator and denominator by \( \cos^6 x \)
\[ \Rightarrow I = \int \frac{\tan^2 x \cdot \sec^2 x}{(1 + \tan^3 x)^2} \, dx \]

Let, \( \tan^3 x = z \)
\[ \Rightarrow 3\tan^2 x \cdot \sec^2 x \, dx = dz \]
\[ I = \frac{1}{3} \int \frac{dz}{z^2} = \frac{-1}{3z} + C \]
\[ = \frac{-1}{3(1 + \tan^3 x)} + C \]

75. Then value of \( \int_{\frac{\pi}{2}}^{\pi} \frac{\sin^2 x}{1 + 2^x} \, dx \) is :
(1) \( \frac{\pi}{8} \) 
(2) \( \frac{\pi}{2} \) 
(3) \( 4\pi \) 
(4) \( \frac{\pi}{4} \)

Answer (4)

Sol. 
\[ \frac{\pi}{2} \int_{-\infty}^{2\sqrt{2}} \frac{\sin^2 x}{1 + 2^x} \, dx \]

Also, 
\[ \frac{\pi}{2} \int_{-2\sqrt{2}}^{\infty} \frac{\sin^2 x}{1 + 2^x} \, dx \]

Adding (i) and (ii)
\[ 2I = \frac{\pi}{2} \int \frac{\sin^2 x}{1 + 2^x} \, dx \]
\[ I = \frac{\pi}{4} \]

76. Let \( g(x) = \cos^2 x, f(x) = \sqrt{x} \), and \( \alpha, \beta (\alpha < \beta) \) be the roots of the quadratic equation \( 18x^2 - 9\pi x + \pi^2 = 0 \). Then the area (in sq. units) bounded by the curve \( y = (gof)(x) \) and the lines \( x = \alpha, x = \beta \) and \( y = 0 \), is
(1) \( \frac{1}{2}(\sqrt{3} - 1) \) 
(2) \( \frac{1}{2}(\sqrt{3} + 1) \) 
(3) \( \frac{1}{2}(\sqrt{3} - \sqrt{2}) \) 
(4) \( \frac{1}{2}(\sqrt{2} - 1) \)

Answer (1)

Sol. \( 18x^2 - 9\pi x + \pi^2 = 0 \)
\[ (6x - \pi)(3x - \pi) = 0 \]
\[ \therefore x = \frac{\pi}{6}, \frac{\pi}{3} \]
78. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

(1) \(3x + 2y = 6\)  
(2) \(2x + 3y = xy\)  
(3) \(3x + 2y = xy\)  
(4) \(3x + 2y = 6xy\)

Answer (3)

Sol. Let the equation of line be \(\frac{x}{a} + \frac{y}{b} = 1\) \ ...(i)

(i) passes through the fixed point (2, 3)

\[\Rightarrow \frac{2}{a} + \frac{3}{b} = 1\] \ ...(ii)

P\((a, 0)\), Q\((0, b)\), O\((0, 0)\). Let \(R(h, k)\),

\[\begin{align*}
Q(0, b) & \quad \text{R}(h, k) \\
O(0, 0) & \quad P(a, 0)
\end{align*}\]

Midpoint of OR is \(\left(\frac{h}{2}, \frac{k}{2}\right)\) \(\Rightarrow h = a, k = b \) \ ...(iii)

From (ii) & (iii),

\[\frac{2}{h} + \frac{3}{k} = 1 \Rightarrow 3x + 2y = xy\]

79. Let the orthocentre and centroid of a triangle be \(A(-3, 5)\) and \(B(3, 3)\) respectively. If \(C\) is the circumcentre of this triangle, then the radius of the circle having line segment \(AC\) as diameter, is

(1) \(10\)  
(2) \(2\sqrt{10}\)  
(3) \(\frac{5}{\sqrt{2}}\)  
(4) \(\frac{3\sqrt{5}}{2}\)

Answer (3)

Sol. \(A(-3, 5)\)

\(B(3, 3)\)
So, \(AB = 2\sqrt{10}\)

Now, as, \(AC = \frac{3}{2}AB\)

So, radius \(= \frac{3}{4}AB = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{15}{2}}\)

80. If the tangent at \((1, 7)\) to the curve \(x^2 = y - 6\) touches the circle \(x^2 + y^2 + 16x + 12y + c = 0\) then the value of \(c\) is

(1) 195  
(2) 185  
(3) 85  
(4) 95

Answer (4)

Sol. Equation of tangent at \((1, 7)\) to curve \(x^2 = y - 6\) is

\[x - 1 = \frac{1}{2}(y + 7) - 6\]

\[2x - y + 5 = 0 \quad ...(i)\]

Centre of circle = \((-8, -6)\)

Radius of circle \(= \sqrt{64 + 36 - c} = \sqrt{100 - c}\)

\[\therefore \text{Line (i) touches the circle}
\]

\[\Rightarrow \frac{2(-8) - (-6) + 5}{\sqrt{4 + 1}} = \sqrt{100 - c}
\]

\[\Rightarrow \sqrt{5} = \sqrt{100 - c}
\]

\[\Rightarrow c = 95\]

81. Tangent and normal are drawn at \(P(16, 16)\) on the parabola \(y^2 = 16x\), which intersect the axis of the parabola at \(A\) and \(B\), respectively. If \(C\) is the centre of the circle through the points \(P\), \(A\) and \(B\) and \(\angle CPB = \theta\), then a value of \(\tan \theta\) is

(1) \(\frac{1}{2}\)  
(2) 2  
(3) 3  
(4) \(\frac{4}{3}\)

Answer (2)

Sol. \(y^2 = 16x\)

Tangent at \(P(16, 16)\) is \(2y = x + 16\) \(\quad ...(1)\)

Normal at \(P(16, 16)\) is \(y = -2x + 48\) \(\quad ...(2)\)

\(i.e., A \) is \((-16, 0)\); \(B\) is \((24, 0)\)

Now, Centre of circle is \((4, 0)\)

Now, \(m_{PC} = \frac{4}{3}\)

82. Tangents are drawn to the hyperbola \(4x^2 - y^2 = 36\) at the points \(P\) and \(Q\). If these tangents intersect at the point \(T(0, 3)\) then the area (in sq. units) of \(\triangle PTQ\) is

(1) 45\sqrt{5}  
(2) 54\sqrt{3}  
(3) 60\sqrt{3}  
(4) 36\sqrt{5}

Answer (1)

Sol. Clearly \(PQ\) is a chord of contact,

\(i.e., \) equation of \(PQ\) is \(T \equiv 0\)

\[\Rightarrow y = -12\]

Solving with the curve, \(4x^2 - y^2 = 36\)

\[\Rightarrow x = \pm 3\sqrt{5}, y = -12\]

\(i.e., \) \(P(3\sqrt{5}, -12); Q(-3\sqrt{5}, -12); T(0, 3)\)

Area of \(\triangle PQT\) is

\[\Delta = \frac{1}{2} \times 6\sqrt{5} \times 15\]

\[= 45\sqrt{5}\]

83. If \(L_1\) is the line of intersection of the planes \(2x - 2y + 3z - 2 = 0, x - y + z + 1 = 0\) and \(L_2\) is the line of intersection of the planes \(x + 2y - z - 3 = 0, 3x - y + 2z - 1 = 0\), then the distance of the origin from the plane containing the lines \(L_1\) and \(L_2\) is

(1) \(\frac{1}{4\sqrt{2}}\)  
(2) \(\frac{1}{3\sqrt{2}}\)  
(3) \(\frac{1}{2\sqrt{2}}\)  
(4) \(\frac{1}{\sqrt{2}}\)

Answer (2)
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Sol. $L_1$ is parallel to
$$\begin{vmatrix}
i & j & k \\
2 & -2 & 3 \\
1 & -1 & 1 \\
\end{vmatrix} = \hat{i} + \hat{j}
$$

$L_2$ is parallel to
$$\begin{vmatrix}
i & j & k \\
1 & 2 & -1 \\
3 & -1 & 2 \\
\end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}
$$

Also, $L_2$ passes through $(\frac{5}{7}, \frac{8}{7}, 0)$

So, required plane is
$$\begin{vmatrix}
x - \frac{5}{7} & y - \frac{8}{7} & z \\
1 & 1 & 0 \\
3 & -5 & -7 \\
\end{vmatrix} = 0$$

Thus, $7x - 7y + 8z + 3 = 0$

Now, perpendicular distance
$$\frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$$

85. Let $\hat{u}$ be a vector coplanar with the vectors $\hat{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{b} = \hat{j} + \hat{k}$. If $\hat{u}$ is perpendicular to $\hat{a}$ and $\hat{u} \cdot \hat{b} = 24$, then $|\hat{u}|^2$ is equal to

(1) 336  (2) 315  (3) 256  (4) 84

Answer (1)

Sol. Clearly, $\hat{u} = \lambda(\hat{a} \times (\hat{a} \times \hat{b}))$

$\Rightarrow \hat{u} = \lambda((\hat{a} \cdot \hat{b})\hat{a} - |\hat{a}|^2 \hat{b})$

$\Rightarrow \hat{u} = \lambda(2\hat{a} - 14\hat{b}) = 2\lambda[(2\hat{i} + 3\hat{j} - \hat{k}) - 7(\hat{j} + \hat{k})]$

$\Rightarrow \hat{u} = 2\lambda(2\hat{i} - 4\hat{j} - 8\hat{k})$

as, $\hat{u} \cdot \hat{b} = 24$

$\Rightarrow 4\lambda(\hat{i} - 2\hat{j} - 4\hat{k}) \cdot (\hat{j} + \hat{k}) = 24$

$\Rightarrow \lambda = -1$

So, $\hat{u} = -4(\hat{i} - 2\hat{j} - 4\hat{k})$

$\Rightarrow |\hat{u}|^2 = 336$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{3}{10}$  (2) $\frac{2}{5}$  (3) $\frac{1}{5}$  (4) $\frac{3}{4}$

Answer (2)

Sol. $E_1$: Event that first ball drawn is red.

$E_2$: Event that first ball drawn is black.

$E$: Event that second ball drawn is red.

$P(E) = P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)$

$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12} = \frac{2}{5}$

Answer (2)
87. If \( \sum_{i=1}^{9}(x_i - 5) = 9 \) and \( \sum_{i=1}^{9}(x_i - 5)^2 = 45 \), then the standard deviation of the 9 items \( x_1, x_2, \ldots, x_9 \) is

(1) 9  (2) 4  (3) 2  (4) 3

Answer (3)

Sol. Standard deviation of \( x_i - 5 \) is

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{9}(x_i - 5)^2}{9} - \left(\frac{\sum_{i=1}^{9}(x_i - 5)}{9}\right)^2}
\]

\( \Rightarrow \sigma = \sqrt{5 - 1} = 2 \)

As, standard deviation remains constant if observations are added/subtracted by a fixed quantity.

So, \( \sigma \) of \( x_i \) is 2

88. If sum of all the solutions of the equation

\[
8 \cos x \cdot \left( \cos \left(\frac{\pi}{6} + x\right) \cdot \cos \left(\frac{\pi}{6} - x\right) - \frac{1}{2} \right) = 1 \quad \text{in } [0, \pi]
\]

is \( k\pi \), then \( k \) is equal to :

(1) \( \frac{2}{3} \)  (2) \( \frac{13}{9} \)  (3) \( \frac{8}{9} \)  (4) \( \frac{20}{9} \)

Answer (2)

Sol. \( 8 \cos x \cdot \left( \cos^2 \frac{\pi}{6} - \sin^2 x - \frac{1}{2} \right) = 1 \)

\( \Rightarrow 8 \cos x \left( \frac{3}{4} - \frac{1}{2} - 1 + \cos^2 x \right) = 1 \)

\( \Rightarrow 8 \cos x \left( -3 + 4 \cos^2 x \right) = 1 \)

\( \Rightarrow \cos 3x = \frac{1}{2} \)

\( \Rightarrow 3x = \frac{\pi}{3}, \frac{7\pi}{3}, \frac{5\pi}{3} \)

\( \Rightarrow x = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{5\pi}{9} \)

\( \Rightarrow \text{Sum} = \frac{13\pi}{9} \)

\( \Rightarrow k = \frac{13}{9} \)

89. PQR is a triangular park with \( PQ = PR = 200 \text{ m} \). A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45º, 30º and 30º, then the height of the tower (in m) is

(1) 100  (2) 50  (3) \( 100\sqrt{3} \)  (4) \( 50\sqrt{2} \)

Answer (1)

Sol.

Let height of tower \( TM \) be \( h \)

\( \therefore PM = h \)

In \( \triangle TQM \),

\( \tan 30^\circ = \frac{h}{QM} \)

\( QM = \sqrt{3} h \)

In \( \triangle PMQ \),

\( PM^2 + QM^2 = PQ^2 \)

\( h^2 + (\sqrt{3} h)^2 = 200^2 \)

\( \Rightarrow 4h^2 = 200^2 \)

\( \Rightarrow h = 100 \text{ m} \)

90. The Boolean expression \( \sim (p \vee q) \vee (\sim p \wedge q) \) is equivalent to

(1) \( \sim p \)  (2) \( p \)  (3) \( q \)  (4) \( \sim q \)

Answer (1)

Sol. \( \sim (p \vee q) \vee (\sim p \wedge q) \)

By property, \( (\sim p \wedge q) \vee (\sim p \wedge q) = \sim p \)