DATE : 07/03/2019

SET-1

Code No. 30/1/1

Aakash
Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Limited)

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Time : 3 Hrs. Max. Marks : 80

Class X

Mathematics

(CBSE 2019)

GENERAL INSTRUCTIONS :

(i) *All questions are compulsory.*

(ii) *This question paper consists of **30** questions divided into four sections - **A, B, C** and **D**.*

(iii) *Section **A** contains **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of **4** marks each.*

(iv) *There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternatives in all such questions.*

(v) *Use of calculator is not permitted.*

Section-A

Question numbers 1 to 6 carry 1 mark each.

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4). **[1]**

Sol. Let the centre be O and coordinates of point A be (x, y)

$\frac{x+1}{2} = 2$ [By Mid-point formula]

$\Rightarrow$ x = 3 [½]

$\frac{y+4}{2} = -3$

$\Rightarrow$ y = –10 [½]

$\therefore$ Coordinates of A = (3, –10)

2. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real? **[1]**

OR

Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

Sol. $x^2 + 4x + k = 0$

$\therefore$ Roots of given equation are real,

$D \geq 0$ **[½]**

$\Rightarrow (4)^2 - 4 \times k \geq 0$

$\Rightarrow -4k \geq -16$

$\Rightarrow k \leq 4$

$\therefore$ k has all real values ≤ 4 **[½]**

OR

$3x^2 - 10x + k = 0$

$\because$ Roots of given equation are reciprocal of each other.

Let the roots be α and $\dfrac{1}{\alpha}$ **[½]**

Product of roots $= \dfrac{c}{a}$

$\Rightarrow \alpha . \dfrac{1}{\alpha} = \dfrac{k}{3}$

$\therefore k = 3$ **[½]**

3. Find A if tan 2A = cot(A – 24°) **[1]**

OR

Find the value of $(\sin^2 33° + \sin^2 57°)$

Sol. $\tan 2A = \cot(A - 24°)$

$\Rightarrow \cot(90° - 2A) = \cot(A - 24°)$ **[½]**

$\Rightarrow 90° - 2A = A - 24°$

$\Rightarrow 3A = 114°$

$\Rightarrow A = 38°$ **[½]**

OR

$\sin^2 33° + \sin^2 57°$

$= \sin^2 33° + \cos^2 (90° - 57°)$ **[½]**

$= \sin^2 33° + \cos^2 33°$

$= 1$ **[½]**

4. How many two digits numbers are divisible by 3? **[1]**

Sol. Two digits numbers divisible by 3 are

12, 15, 18, ..., 99.

$a = 12, d = 15 - 12 = 3$ **[½]**

$\Rightarrow T_n = 99$

$\Rightarrow a + (n - 1)d = 99$

$\Rightarrow 12 + (n - 1)3 = 99$

$\Rightarrow n = 30$

$\therefore$ Number of two digit numbers divisible by 3 are 30. **[½]**

5. In Fig. 1, DE || BC, AD = 1 cm and BD = 2 cm. What is the ratio of the ar(ΔABC) to the ar(ΔADE)? [1]

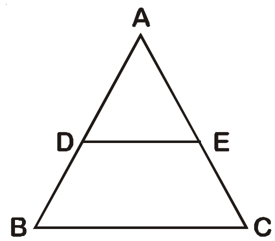
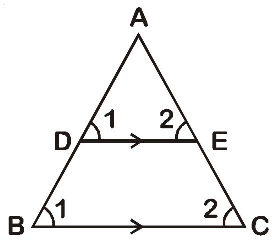


Fig. 1

Sol.



DE || BC

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity] [½]

$$\frac{ar(\Delta ABC)}{ar(ADE)} = \left(\frac{AB}{AD}\right)^2$$ [By area similarity theorem]

$$= \left(\frac{3}{1}\right)^2$$

$$= \frac{9}{1}$$ [½]

6. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$. [1]

Sol. Rational number lying between $\sqrt{2}$ and $\sqrt{3}$ is $1.5 = \frac{15}{10} = \frac{3}{2}$ [½]

[$\because$ $\sqrt{2} \sim 1.414$ and $\sqrt{3} \sim 1.732$] [½]

Section-B

Question numbers 7 to 12 carry 2 marks each.

7. Find the HCF of 1260 and 7344 using Euclid's algorithm. [2]

OR

Show that every positive odd integer is of the form (4q + 1) or (4q + 3), where q is some integer.

Sol. Since 7344 > 1260

$7344 = 1260 \times 5 + 1044$ [½]

Since remainder $\neq 0$

$1260 = 1044 \times 1 + 216$

$1044 = 216 \times 4 + 180$ [½]

$216 = 180 \times 1 + 36$

$180 = 36 \times 5 + 0$ [½]

The remainder has now become zero.

$\therefore$ HCF of 1260 and 7344 is 36. [½]

OR

Let a be positive odd integer

Using division algorithm on a and b = 4 **[½]**

$a = 4q + r$

Since $0 \leq r < 4$, the possible remainders are 0, 1, 2 and 3 **[½]**

$\therefore$ a can be 4q or 4q + 1 or 4q + 2 or 4q + 3, where q is the quotient

Since a is odd, a cannot be 4q and 4q + 2 **[½]**

$\therefore$ Any odd integer is of the form 4q + 1 or 4q + 3, where q is some integer. **[½]**

8. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term? **[2]**

OR

If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term.

Sol. Given AP is

3, 15, 27, 39 ....

where a = 3, d = 15 – 3 = 12 **[½]**

Let the n^{th} term be 120 more than its 21^{st} term.

$t_n = t_{21} + 120$ **[½]**

$\Rightarrow$ $3 + (n - 1)12 = 3 + 20 \times 12 + 120$

$\Rightarrow$ $(n - 1) \times 12 = 363 - 3$ **[½]**

$\Rightarrow$ $(n-1) = \dfrac{360}{12}$

$\therefore$ $n = 31$

Hence, the required term is $t_{31} = 3 + 30 \times 12$

$= 363$ **[½]**

OR

$S_n = 3n^2 - 4n$

Let S_{n-1} be sum of (n – 1) terms

$t_n = S_n - S_{n-1}$ **[½]**

$= (3n^2 - 4n) - [3(n - 1)^2 - 4(n - 1)]$ **[½]**

$= (3n^2 - 4n) - [3n^2 - 6n + 3 - 4n + 4]$ **[½]**

$= 3n^2 - 4n - 3n^2 + 10n - 7$

$\therefore$ $t_n = 6n - 7$

So, required n^{th} term = 6n – 7 **[½]**

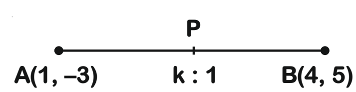
9. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also find the coordinates of this point on x-axis. **[2]**

Sol. Let P(x, y) divides the line segment joining the points A(1, –3) and B(4, 5) internally in the ratio k : 1.

Using section formula, we get

$x = \dfrac{4k+1}{k+1}$ ...(i) and $y = \dfrac{5k-3}{k+1}$ ...(ii) **[½]**

Since, P lies on x-axis. So its ordinate will be zero.



$\Rightarrow$ $\dfrac{5k-3}{k+1} = 0$

$\Rightarrow$ $k = \dfrac{3}{5}$

Hence, the required ratio is 3 : 5. **[½]**

Now putting the value of k in (i) and (ii), we get

$$x = \frac{17}{8} \text{ and } y = 0$$

So, coordinates of point P are $\left(\frac{17}{8}, 0\right)$ [1]

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game. [2]

Sol. Total possible outcomes are (HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT) i.e., 8. [½]

The favourable outcomes to the event E 'Same result in all the tosses' are TTT, HHH. [½]

So, the number of favourable outcomes = 2

$\therefore \quad P(E) = \frac{2}{8} = \frac{1}{4}$ [½]

Hence, probability of losing the game = 1 – P(E)

$$= 1 - \frac{1}{4} = \frac{3}{4}$$ [½]

11. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6. [2]

Sol. Total outcomes = 1, 2, 3, 4, 5, 6 [½]

Prime numbers = 2, 3, 5

Numbers lie between 2 and 6 = 3, 4, 5 [½]

(i) P (Prime Numbers) $= \frac{3}{6} = \frac{1}{2}$ [½]

(ii) P (Numbers lie between 2 and 6) $= \frac{3}{6} = \frac{1}{2}$ [½]

12. Find c if the system of equations cx + 3y + (3 – c) = 0; 12x + cy – c = 0 has infinitely many solutions? [2]

Sol. For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ [½]

$$\begin{array}{ccccc} \text{I} & & \text{II} & & \text{III} \\ \frac{c}{12} & = & \frac{3}{c} & = & \frac{3-c}{-c} \end{array}$$

(i) $c^2 = 12 \times 3$ [From I and II]

$c = \pm 6$ [½]

(ii) $\frac{3}{c} = \frac{3-c}{-c}$ [From II and III]

$-3c = 3c - c^2$

$c^2 - 6c = 0$

$c = 0, 6$

(iii) $c^2 = 12(c - 3)$ [From I and III] [½]

$c^2 - 12c + 36 = 0$

$(c - 6)^2 = 0$

$c = 6$

Hence the value of c is 6. [½]

Section-C

Question numbers 13 to 22 carry 3 marks each.

13. Prove that $\sqrt{2}$ is an irrational number. **[3]**

Sol. Let $\sqrt{2}$ be rational. Then, there exist positive integers a and b such that $\sqrt{2}=\frac{a}{b}$, where a and b are co-prime, $b \neq 0$ **[½]**

$\Rightarrow \quad (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$ **[½]**

$\Rightarrow \quad 2 = \frac{a^2}{b^2}$

$\Rightarrow \quad 2b^2 = a^2$

$\therefore \quad$ 2 divides a^2

$\Rightarrow \quad$ 2 divides a ...(i)

Let a = 2c for some integer c **[½]**

$a^2 = 4c^2$

$\Rightarrow \quad 2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2$

$\therefore \quad$ 2 divides b^2

$\Rightarrow \quad$ 2 divides b ...(ii) **[½]**

From (i) and (ii), we get

$\therefore \quad$ 2 is common factor of both a and b.

But this contradicts the fact that a and b have no common factor other than 1 **[½]**

$\therefore \quad$ Our supposition is wrong

Hence, $\sqrt{2}$ is an irrational number. **[½]**

14. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half of their product. **[3]**

Sol. For given polynomial

$x^2 - (k + 6)x + 2(2k - 1)$, **[½]**

Let the zeroes be α & β.

So, $\alpha+\beta = -\frac{b}{a} = k+6, \quad \alpha\beta = \frac{c}{a} = \frac{4k-2}{1}$ **[1]**

$\because \quad$ Sum of zeroes = $\frac{1}{2}$ (product of zeroes)

$\Rightarrow \quad \alpha+\beta = \frac{1}{2}\alpha\beta$ **[½]**

$\Rightarrow \quad k+6 = \frac{1}{2}(4k-2)$

$\Rightarrow$ $k + 6 = 2k - 1$

$\therefore$ $k = 7$

So, the value of k is 7 **[1]**

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father. **[3]**

OR

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

Sol. Let the present age of father be x years and the sum of present ages of his two children be y years. **[½]**

According to question

$x = 3y$ **[½]**

$\Rightarrow$ $x - 3y = 0$ ...(1)

After 5 years,

$x + 5 = 2(y + 10)$

$\Rightarrow$ $x - 2y = 15$ ...(2) **[½]**

On subtracting equation (1) from (2), we get:

$$\begin{array}{rcrcr} x & - & 2y & = & 15 \\ x & - & 3y & = & 0 \\ - & & + & & - \\ \hline & & y & = & 15 \end{array}$$ **[1]**

On substituting the value of y = 15 in (1), we get:

$x - 3 \times 15 = 0$

$\therefore$ $x = 45$ **[½]**

Hence, the present age of father is 45 years.

OR

Let the numerator of required fraction be x and the denominator of required fraction be y ($y \neq 0$)

According to question; **[½]**

$$\frac{x-2}{y} = \frac{1}{3}$$

$\Rightarrow$ $3x - 6 = y$ **[½]**

$\Rightarrow$ $3x - y = 6$ ...(1)

and

$$\frac{x}{y-1} = \frac{1}{2}$$

$\Rightarrow$ $2x = y - 1$ **[½]**

$\Rightarrow$ $2x - y = -1$ ...(2)

On subtracting (2) from (1), we get:

$$\begin{array}{rcrcr} 3x & - & y & = & 6 \\ 2x & - & y & = & -1 \\ - & & + & & + \\ \hline & & x & = & 7 \end{array}$$ [1]

On substituting x = 7 in (1), we get:

$3(7) - y = 6$

$\Rightarrow \quad -y = 6 - 21$

$\therefore \quad y = 15$ [½]

Hence, the required fraction is $\frac{x}{y} = \frac{7}{15}$.

16. Find the point on y-axis which is equidistant from the points (5, – 2) and (– 3, 2). [3]

OR

The line segment joining the points A(2, 1) and B(5, – 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k.

Sol. Let the point on y-axis be P(0, y) which is equidistant from the points A(5, –2) and B(– 3, 2). [½]

We are given that AP = BP

So, $AP^2 = BP^2$ [½]

i.e., $(5 - 0)^2 + (-2 - y)^2 = (-3 - 0)^2 + (2 - y)^2$ [1]

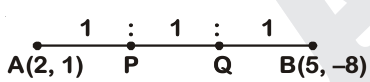
$\Rightarrow \quad 25 + y^2 + 4 + 4y = 9 + 4 + y^2 - 4y$

$\Rightarrow \quad 8y = -16$

$\Rightarrow \quad y = -2$ [1]

Hence, the required point is (0, – 2)

OR



Here, AP : PB = 1 : 2 [½]

$$\therefore \quad P \equiv \left(\frac{1\times5+2\times2}{1+2}, \frac{1\times-8+2\times1}{1+2}\right)$$

$\Rightarrow \quad P \equiv (3, -2)$ [1]

Since, P lies on the line $2x - y + k = 0$ [½]

$\therefore \quad 2(3) - (-2) + k = 0$

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8$ [1]

17. Prove that : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$. [3]

OR

Prove that : $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.

Sol. L.H.S : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2 \qquad \left[\because \sin\theta = \frac{1}{\operatorname{cosec}\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$ [1]

$= \sin^2\theta + \cos^2\theta + 1 + \cot^2\theta + 1 + \tan^2\theta + 4$ [$\because$ $\text{cosec}^2\theta + 1 + \cot^2\theta$ and $\sec^2\theta = 1 + \tan^2\theta$] **[1]**

$= 1 + 1 + 1 + 4 + \tan^2\theta + \cot^2\theta$ [$\because$ $\cos^2\theta + \sin^2\theta = 1$] **[½]**

$= 7 + \tan^2\theta + \cot^2\theta$

L.H.S = R.H.S **[½]**

OR

$$\text{L.H.S}: \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$=\left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$ **[½]**

$$=\frac{(\sin A+\cos A)^2-(1)^2}{\sin A\cdot\cos A}$$ **[½]**

$$=\frac{\sin^2 A+\cos^2 A+2\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$$ **[½]**

$$=\frac{1+2\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$$ [$\because$ $\sin^2A + \cos^2A = 1$] **[½]**

$= 2$

Hence, L.H.S = R.H.S **[1]**

18. In Fig. 2, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP. **[3]**

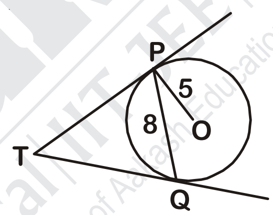
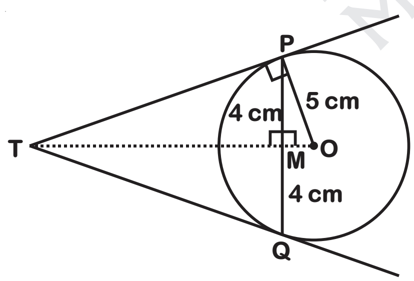


Fig. 2

Sol. Join OT which bisects PQ at M and perpendicular to PQ



In ΔOPM,

$OP^2 = PM^2 + OM^2$ [By pythagoras Theorem]

$\Rightarrow (5)^2 = (4)^2 + OM^2$

$\Rightarrow OM = 3$ cm **[1]**

In ΔOPT and ΔOPM,

$\angle MOP = \angle TOP$ [Common angles]

$\angle OMP = \angle OPT$ [Each 90°]

$\therefore \Delta POT \sim \Delta MOP$ [By AA similarity] [1]

$\Rightarrow \frac{TP}{MP} = \frac{OP}{OM}$

$\Rightarrow TP = \frac{4 \times 5}{3}$ [$\because OP = 5$ cm, $PM = 4$ cm, $MO = 3$ cm]

$\Rightarrow TP = \frac{20}{3} = 6\frac{2}{3}$ cm [1]

19. In Fig. 3, $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$. [3]

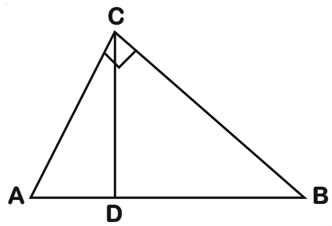
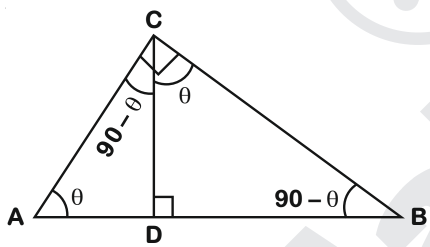


Fig. 3

OR

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

Sol.



Let $\angle A = \theta$

$\therefore$ $\angle ACD = 90 - \theta$, $\angle BCD = \theta$, $\angle CBD = 90 - \theta$ [½]

$\because$ $\angle CAD = \angle BCD$

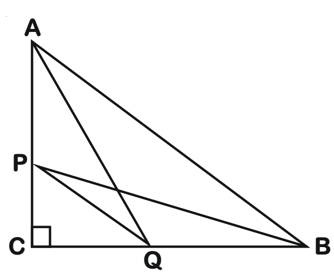
and $\angle ACD = \angle CBD$ [½]

$\Delta CAD \sim \Delta BCD$ [By AA similarity] [1]

$\therefore$ $\frac{AD}{CD} = \frac{CD}{BD}$ [½]

$\therefore$ $CD^2 = AD \times BD$ [½]

OR



In right ΔACQ,

$AQ^2 = AC^2 + CQ^2$ ...(i) [By Pythagoras theorem] [1]

In right ΔPCB,

$BP^2 = PC^2 + CB^2$ ...(ii) [By Pythagoras theorem] [1]

On adding equations (i) and (ii), we get

$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + CB^2$ [½]

$= (AC^2 + CB^2) + (CQ^2 + PC^2)$

$= AB^2 + PQ^2$ [By Pythagoras theorem] [½]

20. Find the area of the shaded region in Fig. 4, if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take π = 3.14) [3]

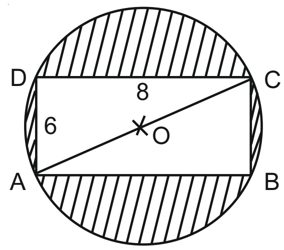
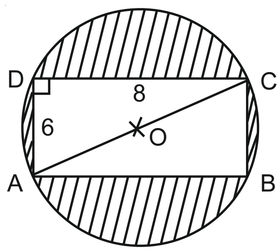


Fig. 4

Sol.



In right triangle ADC, $\angle D = 90°$

$AC^2 = AD^2 + DC^2$ [By Pythagoras theorem] [½]

$= 6^2 + 8^2 = 100$

AC = 10 cm [½]

2(AO) = 10

AO = 5 cm

$\Rightarrow$ Radius (r) = 5 cm [½]

Area of the shaded region = Area of the circle – Area of rectangle [½]

$= \pi r^2 - l \times b$

$= 3.14(5)^2 - 6 \times 8$ [½]

$= 78.5 - 48 = 30.5\ cm^2$ [½]

21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed? [3]

Sol. Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of the water column formed in $\frac{1}{2}$ hr

= 5 km or 5000 m [½]

$\therefore$ Volume of water flowing in $\frac{1}{2}$ hr

= Volume of cuboid of length 5000 m, width 6 m and depth 1.5 m.

$= 5000 \times 6 \times 1.5 = 45000\ m^3$ [1]

On comparing the volumes,

Volume of water in field = Volume of water coming out from canal in 30 minutes. [½]

Irrigated area × standing water = 45000.

Irrigated Area = $\dfrac{45000}{\dfrac{8}{100}}$ [$\because$ 1 m = 100 cm] [½]

$= \dfrac{45000 \times 100}{8}$ = 5,62,500 m^3 [½]

22. Find the mode of the following frequency distribution. [3]

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	8	10	10	16	12	6	7

Sol. [½]

Class	Frequency
0 - 10	8
10 - 20	10
20 - 30	$10 \to f_0$
30 - 40	$16 \to f_1$
40 - 50	$12 \to f_2$
50 - 60	6
60 - 70	7

Here, 30 - 40 is the modal class, and l = 30, h = 10 [½]

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ [1]

$= 30 + \left(\dfrac{16 - 10}{2 \times 16 - 10 - 12}\right) \times 10$ [½]

$= 30 + \dfrac{6}{10} \times 10$ = 30 + 6 = 36 [½]

Section-D

Question numbers 23 to 30 carry 4 marks each.

23. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately. [4]

OR

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Sol. Let the time in which tap with longer and smaller diameter can fill the tank separately be x hours and y hours respectively. [½]

According to the question

$$\frac{1}{x}+\frac{1}{y}=\frac{8}{15} \quad \text{...(i)} \quad [½]$$

and x = y – 2 ...(ii) [½]

On substituting x = y – 2 from (ii) in (i), we get

$$\frac{1}{y-2}+\frac{1}{y}=\frac{8}{15} \quad [½]$$

$$\Rightarrow \frac{y+y-2}{y^2-2y}=\frac{8}{15}$$

$\Rightarrow$ $15(2y - 2) = 8(y^2 - 2y)$

$\Rightarrow$ $30y - 30 = 8y^2 - 16y$

$\Rightarrow$ $8y^2 - 46y + 30 = 0$ [½]

$\Rightarrow$ $4y^2 - 20y - 3y + 15 = 0$

$\Rightarrow$ $(4y - 3)(y - 5) = 0$

$$\Rightarrow y=\frac{3}{4}, y=5 \quad [½]$$

Substituting values of y in (ii), we get

$x=\frac{3}{4}-2$	$x=5-2$	[½]
$x=\frac{-5}{4}$	$x=3$	
$\therefore x \neq \frac{-5}{4}$		
(time cannot be negative)		

Hence, the time taken by tap with longer diameter is 3 hours and the time taken by tap with smaller diameter is 5 hours, in order to fill the tank separately. [½]

OR

Let the speed of the boat in still water be x km/h and speed of the stream be y km/h. [½]

According to question,

$$\frac{30}{x-y}+\frac{44}{x+y}=10 \quad \text{...(i)}$$

$$\text{and } \frac{40}{x-y}+\frac{55}{x+y}=13 \quad \text{...(ii)} \quad [½]$$

Let $\frac{1}{x-y}=a$

and $\frac{1}{x+y}=b$, then we get

$30a + 44b = 10$ ... (iii)

$40a + 55b = 13$ ...(iv) [½]

On solving (iii) and (iv), we get

$$\begin{array}{r} 120a + 176b = 40 \\ 120a + 165b = 39 \\ - \quad - \quad - \\ \hline 11b = 1 \end{array}$$ [½]

$\Rightarrow \quad b = \dfrac{1}{11}$

Substituting $b = \dfrac{1}{11}$ in (iii), we get

$$30a + 44 \times \frac{1}{11} = 10$$

$\Rightarrow \quad a = \dfrac{1}{5}$ [½]

$\because \quad a = \dfrac{1}{5}$ & $b = \dfrac{1}{11}$

$\Rightarrow \quad x - y = 5$ ...(v)

and $x + y = 11$ ...(vi)

Adding (v) and (vi), we get [½]

$$\begin{array}{r} x - y = 5 \\ x + y = 11 \\ \hline 2x = 16 \end{array}$$

$\therefore \quad x = 8$ [½]

Substituting $x = 8$ in (vi), we get

$8 + y = 11$

$\therefore \quad y = 3$

Hence, speed of the stream is 3 km/h & speed of the boat in still water is 8 km/h. [½]

24. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms. [4]

Sol. Let the first four terms be a, a + d, a + 2d, a + 3d

$a + a + d + a + 2d + a + 3d = 40$ [½]

$\Rightarrow \quad 2a + 3d = 20$ ...(i) [½]

Sum of first 14 terms = 280

$$\frac{n}{2}[2a + (n-1)d] = 280$$ [½]

$$\Rightarrow \quad \frac{14}{2}[2a + 13d] = 280$$

$\Rightarrow \quad 2a + 13d = 40$ ...(ii) [1]

On subtracting (i) from (ii), we get d = 2

Put the value of d in (i) [½]

a = 7

$\therefore$ Sum of n terms $= \frac{n}{2}[2a+(n-1)d]$ [½]

$= \frac{n}{2}[14+(n-1)2]$

$= n^2 + 6n$ [½]

25. Prove that $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$ [4]

Sol. LHS $= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$ [½]

$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$ (Dividing numerator & denominator by cos A) [½]

$= \frac{(\tan A + \sec A) - 1}{(\tan A - \sec A) + 1}$ [½]

$= \frac{\{(\tan A + \sec A) - 1\}(\tan A - \sec A)}{\{(\tan A - \sec A) + 1\}(\tan A - \sec A)}$ [½]

$= \frac{(\tan^2 A - \sec^2 A) - (\tan A - \sec A)}{\{\tan A - \sec A + 1\}(\tan A - \sec A)}$ [½]

$= \frac{-1 - \tan A + \sec A}{(\tan A - \sec A + 1)(\tan A - \sec A)}$ [½]

$= \frac{-1}{\tan A - \sec A}$ [½]

$= \frac{1}{\sec A - \tan A}$ [½]

LHS = RHS

Hence Proved.

26. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$] [4]

OR

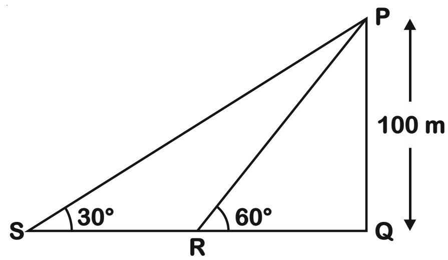
Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Sol. Let the tower be PQ and the boat changes its position from R to S.

Here, PQ = 100 m, $\angle PRQ = 60°$ and $\angle PSR = 30°$.

In ΔPQR,

$$\tan 60^\circ = \frac{PQ}{QR} = \frac{100}{QR}$$



$\Rightarrow \quad QR = \dfrac{100\sqrt{3}}{3}$ m ...(i) [1]

In ΔPQS,

$$\tan 30^\circ = \frac{PQ}{QS}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{QS}$$

$\Rightarrow \quad QS = 100\sqrt{3}$ m [1]

$\therefore \quad RS = QS - QR = 100\sqrt{3} - \dfrac{100\sqrt{3}}{3} = \dfrac{200\sqrt{3}}{3}$ [1]

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{200\sqrt{3}}{3 \times 2} = \frac{100\sqrt{3}}{3}$$

= 57.73 (approx.) (Using $\sqrt{3} = 1.732$)

= 57.73 m/min [1]

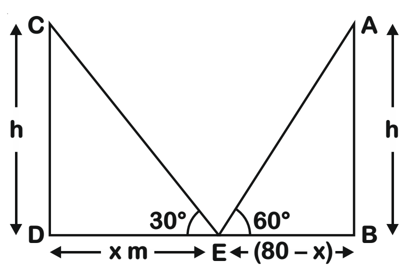
OR

Let the poles be AB, CD each of height h meter and E is the point between the poles on the road.

Let $\angle AEB = 60°$, $\angle CED = 30°$ and DE be x meter. [½]

$\therefore \quad BE = (80 - x)$ m

In ΔAEB,



$\tan 60^\circ = \dfrac{AB}{BE}$ [½]

$\Rightarrow \quad \sqrt{3} = \dfrac{h}{(80 - x)}$ [½]

$\Rightarrow \quad h = \sqrt{3}(80 - x)$ m ...(i)

In ΔCDE,

$\tan 30^\circ = \dfrac{CD}{DE}$ [½]

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$\Rightarrow \quad h = \frac{x}{\sqrt{3}}$ m ...(ii) [½]

From equation (i) and (ii), we get

$$\frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$$ [½]

$\Rightarrow \quad x = 240 - 3x$

$\Rightarrow \quad 4x = 240$

$\Rightarrow \quad x = 60$ m [½]

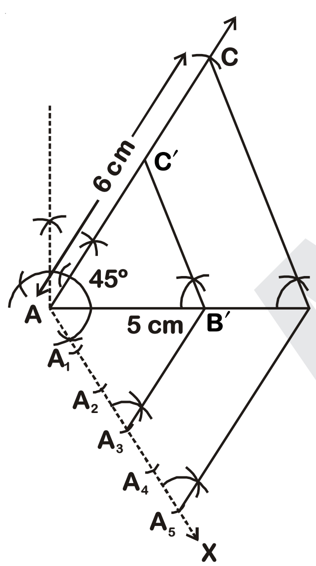
Put value of x in equation (ii), we get

$$h = 20\sqrt{3} \text{ m}, \ DE = 60 \text{ m} \text{ and } BE = 20 \text{ m}$$

Hence, the heights of each pole is $20\sqrt{3}$ m and distance of the point from the poles are 60 m and 20 m. [½]

27. Construct a ΔABC in which CA = 6 cm, AB = 5 cm and $\angle BAC = 45°$. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC. [4]

Sol.



[2½]

Steps of construction :

(i) Construct ΔABC such that AB = 5 cm, $\angle CAB = 45°$ and CA = 6 cm.

(ii) Draw any ray AX making an acute angle with AB on the side opposite to the vertex C.

(iii) Mark points A_1, A_2, A_3, A_4, A_5 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

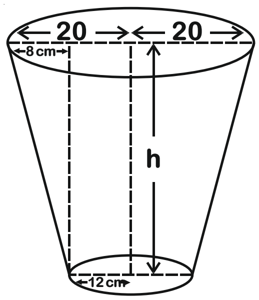
(iv) Join A_5B.

(v) Through A_3, draw a line parallel to A_5B intersecting with AB at B′.

(vi) Through B′, draw a line parallel to BC intersecting with AC at C′.

Now, $\Delta AB'C'$ is the required triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC. [1½]

28. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14) **[4]**

Sol. 

Let the height of the bucket be h cm and slant height be l cm.

Here r_1 = 20 cm

r_2 = 12 cm **[½]**

And capacity of bucket = 12308.8 cm^3

We know that capacity of bucket $=\frac{\pi h}{3}(r_1^2+r_2^2+r_1r_2)$ **[½]**

$$=3.14\times\frac{h}{3}[400+144+240]$$

$$=3.14\times\frac{h}{3}\times784$$

So we have $\frac{h}{3}\times3.14\times784=12308.8$ **[½]**

$$h=\frac{12308.8\times3}{3.14\times784}$$

= 15 cm **[½]**

Now, the slant height of the frustum,

$l = \sqrt{h^2+(r_1-r_2)^2}$ **[½]**

$= \sqrt{15^2+8^2}$

$= \sqrt{289}$ **[½]**

= 17 cm

Area of metal sheet used in making it

$= \pi r_2^2 + \pi(r_1 + r_2)l$ **[½]**

= 3.14 × [144 + (20 + 12) × 17]

= 2160.32 cm^2 **[½]**

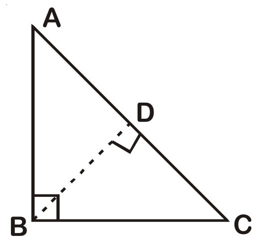
29. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides. **[4]**

Sol. **Given :** A right triangle ABC in which $\angle B = 90°$

To Prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e. $AC^2 = AB^2 + BC^2$ **[½]**

Construction : From B, draw $BD \perp AC$



[½]

Proof : In ΔABC and ΔADB

$\angle BAC = \angle DAB$ [Common]

$\angle ABC = \angle ADB$ [Each 90°]

$\therefore$ $\Delta ABC \sim \Delta ADB$ [By AA similarity] [½]

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$

$\Rightarrow AB^2 = AD \times AC$ ...(i) [½]

Similarly, $\Delta ABC \sim \Delta BDC$ [½]

$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC}$

$\Rightarrow BC^2 = AC \times DC$ ...(ii) [½]

On Adding (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$ [½]

$\Rightarrow AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\therefore$ $AC^2 = AB^2 + BC^2$ [½]

30. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2. [4]

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

OR

The marks obtained by 100 students of a class in an examination are given below.

Marks	No. of Students
0 – 5	2
5 – 10	5
10 – 15	6
15 – 20	8
20 – 25	10
25 – 30	25
30 – 35	20
35 – 40	18
40 – 45	4
45 – 50	2

Draw ‘a less than’ type cumulative frequency curves (ogive). Hence find median.

Sol. [1]

Class	Frequency	Cumulative Frequency
0 – 10	f_1	f_1
10 – 20	5	$5 + f_1$
20 – 30	9	$14 + f_1$
30 – 40	12	$26 + f_1$
40 – 50	f_2	$26 + f_1 + f_2$
50 – 60	3	$29 + f_1 + f_2$
60 – 70	2	$31 + f_1 + f_2$
Total = 40 = n		

$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$f_1 + f_2 = 40 - 31 = 9$ ...(i) [½]

Median = 32.5 [Given]

$\therefore$ Median Class is 30 – 40

$\ell = 30$, h = 10, cf = $14 + f_1$, f = 12 [½]

$$\text{Median} = \ell + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$ [½]

$$32.5 = 30 + \left[\frac{\frac{40}{2} - (14 + f_1)}{12}\right] \times 10$$ [½]

$$2.5 = \frac{10}{12}(20 - 14 - f_1)$$

$3 = 6 - f_1$

$f_1 = 3$ [½]

On putting in (i),

$f_1 + f_2 = 9$

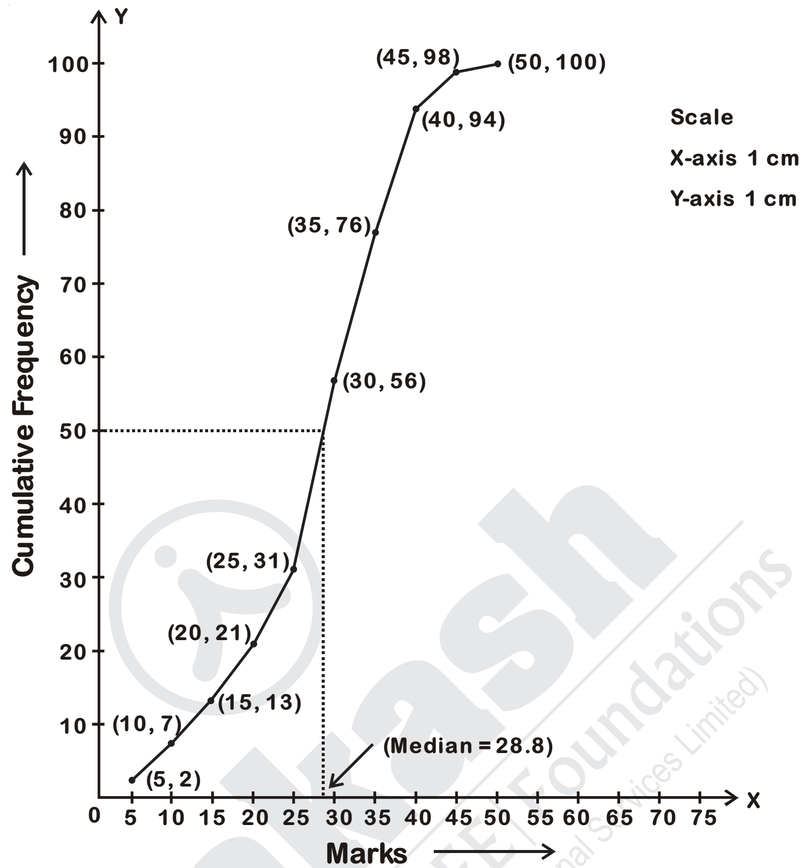
$f_2 = 9 - 3$ [$\because f_1 = 3$] [½]

$= 6$

OR

Marks	No. of students	Marks less than	Cumulative frequency
0-5	2	less than 5	2
5-10	5	less than 10	7
10-15	6	less than 15	13
15-20	8	less than 20	21
20-25	10	less than 25	31
25-30	25	less than 30	56
30-35	20	less than 35	76
35-40	18	less than 40	94
40-45	4	less than 45	98
45-50	2	less than 50	100

Let us now plot the points corresponding to the ordered pairs (5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98), (50, 100). Join all the points by a smooth curve.



Locate $\frac{n}{2} = \frac{100}{2} = 50$ on Y-axis

From this point draw a line parallel to X-axis cutting the curve at a point. From this point, draw a perpendicular to X-axis. The point of intersection of perpendicular with the X-axis determines the median of the data.

Therefore median = 28.8

❑ ❑ ❑